MATHEMATICS

STD - $\overline{X}$
class-72

Assignment

1) The coordinates of the vertices of a triangle are $(1,2)$, $(2,3)$, $(3,1)$. Find the coordinates of the centre of its circumcircle and the circum radius.

Ans) If the circumcentre is $O(x,y)$ and the given points are $A(1,2)$, $B(2,3)$ and $C(3,1)$ then circumradius

$$OA = OB = OC$$

AS $OA^2 = OB^2 = OC^2$

$$OA^2 = (x-1)^2 + (y-2)^2 = x^2 + y^2 - 2x - 4y + 5$$
$$OB^2 = (x-2)^2 + (y-3)^2 = x^2 + y^2 - 4x - 6y + 13$$
$$OC^2 = (x-3)^2 + (y-1)^2 = x^2 + y^2 - 6x - 2y + 10$$

Since $OA^2 = OB^2$

$$x^2 + y^2 - 2x - 4y + 5 = x^2 + y^2 - 4x - 6y + 13$$

$$\therefore -2x - 4y + 5 = -6x - 2y + 13$$

$$2x + 2y = 8 \longrightarrow ①$$

As $OB^2 = OC^2$

$$x^2 + y^2 - 4x - 6y + 13 = x^2 + y^2 - 6x - 2y + 10$$

$$\therefore -4x - 6y + 13 = -6x - 2y + 10$$

$$2x - 4y = -3 \longrightarrow ②$$

From ① and ②

$$① - ②, \quad 6y = 11, \quad y = \frac{11}{6}$$

From ①, $2x = 8 - 2y = 8 - 2 \times \frac{11}{6} = \frac{48 - 22}{6}$

$$2x = \frac{26}{6}, \quad x = \frac{26}{2 \times 6} = \frac{13}{6}$$

$\therefore$ Circumcentre is $\left(\frac{13}{6}, \frac{11}{6}\right)$

Circumradius $= OA = \sqrt{\left(\frac{13}{6}-1\right)^2 + \left(\frac{11}{6}-2\right)^2}$

$$= \sqrt{\left(\frac{13-6}{6}\right)^2 + \left(\frac{11-12}{6}\right)^2}$$

$$= \sqrt{\left(\frac{7}{6}\right)^2 + \left(\frac{-1}{6}\right)^2}$$

$$= \sqrt{\frac{49}{36} + \frac{1}{36}} = \sqrt{\frac{50}{36}}$$

$$= \sqrt{\frac{25 \times 2}{36}}$$

Circumradius $= \dfrac{5\sqrt{2}}{6}$