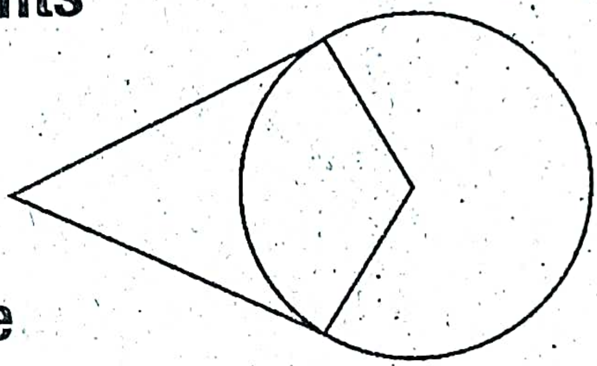
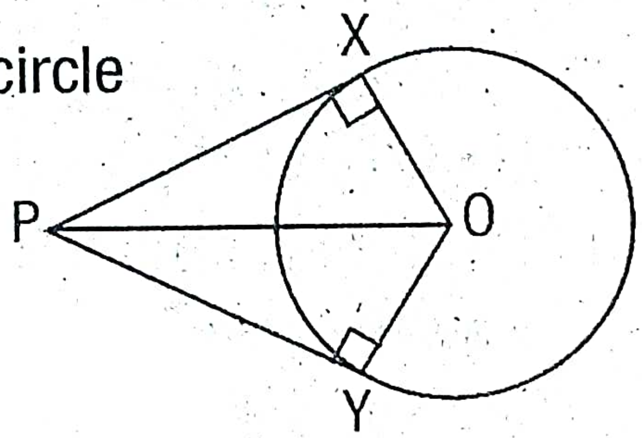
3. The picture shows the tangents at two points on a circle and the radii through the points of contact.



(i) Prove that the tangents have the same length.

OX and OY are two radii of a circle with centre O. PX and PY are tangents to the circle.



We have to prove that PX = PY. Draw OP.

Consider ΔOPX and ΔOPY.

$\angle OXP = \angle OYP = 90^\circ$ (The tangent at a point on a circle is perpendicular to the radius through that point.)

OP = OP (common side)

OX = OY (radii of the same circle.)

The hypotenuse and one side of right triangle OPX is equal to the hypotenuse and one side of right triangle OPY. So these triangles are equal.

In equal triangles, sides opposite to equal angles are equal.

So PX = PY.

Another method

Using Pythagoras principle in right triangle PXO and right triangle PYO, $PO^2 = PX^2 + OX^2$ ...... (1)

$PO^2 = PY^2 + OY^2$ .... (2)

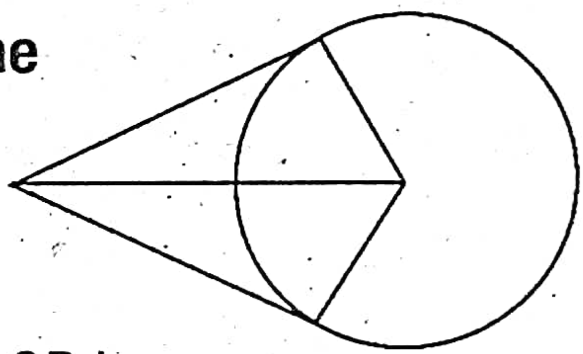
From (1) and (2), $PX^2 + OX^2 = PY^2 + OY^2$

But OX = OY (radii of the same circle)

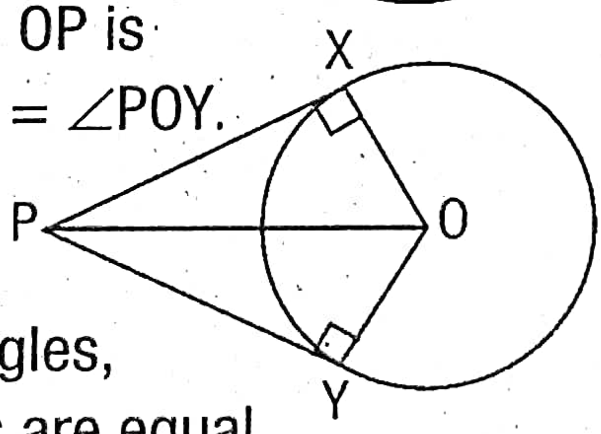
$\therefore PX^2 + OX^2 = PY^2 + OX^2$

$\therefore PX^2 = PY^2$, PX = PY

(ii) Prove that the line joining the centre and the point where the tangents meet bisects the angle between the radii.



We have to prove that the line OP is the bisector of $\angle XOY$ or $\angle POX = \angle POY$.

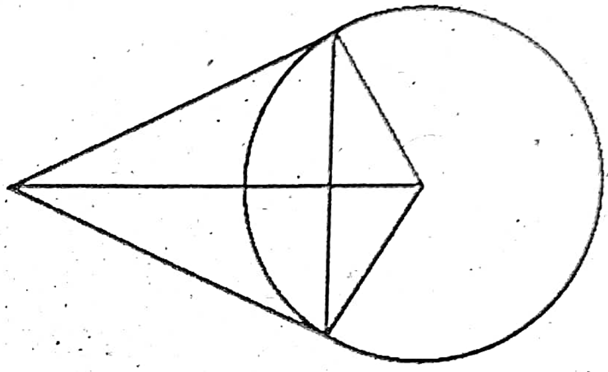


We have proved in (i) that right triangles ΔOPX and ΔOPY are equal. In equal triangles, angles opposite to equal sides are equal.

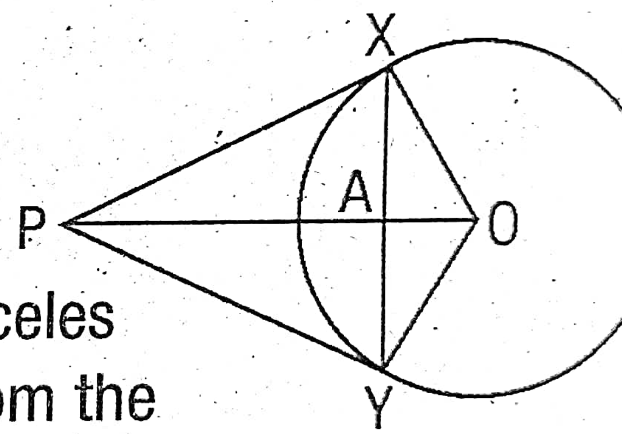
$\therefore \angle POX = \angle POY$

That means the line OP is the bisector of $\angle XOY$.

(iii) Prove that this line is the perpendicular bisector of the chord joining the points of contact.



We have to prove that the line OP is the perpendicular bisector of the chord XY.



Since OX = OY, ΔOXY is an isosceles triangle. In an isosceles triangle, the perpendicular from the vertex to the base bisects the base.

So the line OA is the perpendicular bisector of line XY.