CHAPTER-1 SETS

SET

Set is a well-defined collection of distinct object

* Well-defined : If given any objects, it should be possible to tell whether it is in the collection or not.

* Objects belonging to the set are known as elements or members of the set.

* Sets are denoted by capital letters.

EXAMPLES FOR SETS

(i) The collection of all states in India.
(ii) The collection of all vowels in the English alphabet.
(iii) The collection of CBSE schools in India.

EXAMPLES WHICH ARE NOT SETS

(i) The collection of all interesting books in a library.
(ii) The collection of rich families in India.
(iii) The collection of all tall people in India.

Q. Which of the following are sets

(a) The collection of all states in India beginning with the letter K [✓]

(b) The collection of ten most talented writers in India [x]

(c) The team of eleven best cricket batsmen of the world [×]

(d) The collection of books written by Rabindranath Tagore [✓]

(e) The collection of all odd integers [✓]

(f) The collection of most dangerous animals of the world [×]

(g) The collection of all gold medalists in the olympics held in China [✓]

(h) The collection of all good movies acted by Amitabh Bachchan [×]

(i) The collection of all movies acted by Amitabh Bachchan [✓]

NUMBER SYSTEM

① Natural Numbers [N]

$N = \{1, 2, 3, 4, \ldots\}$

② Whole Numbers [W]

$W = \{0, 1, 2, 3, 4, \ldots\}$

③ Integers [Z / I]

$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
$= \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

④ Rational Numbers [Q]

A number which can be written of the form p/q, $q \neq 0$, where p & q are coprimes. $Q = R - T$

(5) Irrational Numbers [T] $T = R - Q$

(6) Real Numbers [R] $R = Q \cup T$

(7) Complex Numbers [C]

REPRESENTATION OF SETS

Set-builder Form OR Rule Form	Roster Form OR Tabular Form
A = The collection of all natural numbers between 3 & 7 OR $A = \{x : x \text{ is a natural number}, 3 < x < 7\}$ OR $A = \{x : x \in N, 3 < x < 7\}$	$A = \{4, 5, 6\}$ Here $4 \in A$ ↓ belongs to but $7 \notin A$ ↓ not belong to

* In roster form, the order in which the elements are listed is <u>immaterial</u>
* In roster form, we just list all of element separated by commas between them and enclosed in brackets [braces].
* In roster form, elements are generally <u>not repeated</u>.
  Eg $B = \{x : x \text{ is a letter in the word MALAYALAM}\}$
  $= \{M, A, L, Y\}$

  Here $M \in B$, $A \in B$, $L \in B$, $N \notin B$

[redacted]

Q_2 Consider the ~~following~~ set $A = \{5, 10, 15, 20, 25, 30\}$. Insert the appropriate symbol $\in$ / $\notin$ in the blank spaces.

(i) ~~$5 \notin$~~ $5 \underline{\in} A$ (ii) $7 \underline{\notin} A$ (iii) $15 \underline{\in} A$

(iv) $20 \underline{\in} A$ (v) $12 \underline{\notin} A$ (vi) $22 \underline{\notin} A$

(vii) $25 \underline{\in} A$ (viii) $30 \underline{\in} A$

Q_3 Write the following sets in roster form.

(a) $A = \{x : x \text{ is an integer and } -5 < x < 2\}$

Solution

$$A = \{-4, -3, -2, -1, 0, 1\}$$

(b) $B = \{x : x \text{ is a natural number less than } 10\}$

Solution

$$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

(c) $C = \{x : x \text{ is a two digit +ve number such that the sum of its digits is } 7\}$

Solution

$$C = \{16, 25, 34, 43, 52, 61, 70\}$$

(d) $D = \{x : x \text{ is a prime number which is a divisor of } 72\}$

Solution

$$D = \{2, 3\}$$

(e) E = {x : x is a woman president of India}

Solution

$E = \{\text{Pratibha Patel}\}$

(f) F = The set of all letter of the word INDEPENDENCE

Solution

$F = \{I, N, D, E, P, C\}$

(g) G = The set of all letter of the word SAUDI ARABIA

Solution

$G = \{S, A, U, D, I, R, B\}$

(h) H = The solution set of the equation $x^2 - 5x + 6 = 0$

Solution

$$x^2 - 5x + 6 = 0$$
$$x^2 - 3x - 2x + 6 = 0$$
$$x(x-3) - 2(x-3) = 0$$
$$(x-2)(x-3) = 0$$
$$x = 2, 3$$
$$H = \{2, 3\}$$

Q. Write the following sets in the set builder form.

(a) $A = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$

Solution

$A = \{x : x = 3n, n < 11 \text{ and } n \in N\}$

(b) $B = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

Solution

$B = \{x : x = n^2, n \leq 10 \ \& \ n \in N\}$

(c) $C = \{2, 4, 8, 16, 32, 64, 128, 256, 512\}$

Solution

$C = \{x : x = 2^n, n \leq 9 \ \& \ n \in N\}$

(d) $D = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\}$

Solution

$D = \{x : x = \frac{n}{n+1}, n \in N\}$

(e) $E = \{1, 8, 27, 64, 125, 216, 343, 512\}$

Solution

$E = \{x : x = n^3, n \leq 8 \ \& \ n \in N\}$

Q_5 List all the elements of the following sets

(a) $A = \{x : x$ is an odd natural number less than $20\}$

Solution

$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

(b) $B = \{x : x \in Z, -\frac{5}{2} < x < \frac{11}{2}\}$

Solution

$-\frac{5}{2} = -2.5$

$B = \{-2, -1, 0, 1, 2, 3, 4, 5\}$

$\frac{11}{2} = 5.5$

(c) $C = \{x : x \text{ is a month of an year having 31 days}\}$

Solution

$C = \{$ January, March, May, July, August, October, December $\}$

(d) $D = \{x : x \text{ is an Integer}, x^2 \leq 9\}$

Solution

$D = \{-3, -2, -1, 0, 1, 2, 3\}$

ASSIGNMENT

1\. (a) Yes

It is a well defined collection of objects

(b) No

~~A writer~~ Determining a writer's talent vary from person to person.

(c) No

Determining a batsman's talent vary from person to person.

(d) Yes

It is a well defined collection of objects

(e) Yes

It is a well defined collection of objects.

(f) Yes

It is a well defined collection of objects

(g) Yes

It is a well defined collection of objects.

(b) Yes

It is a well defined collection of distinct object

(ii) No.

Determining the dangerousness of an animal vary from person to person.

2. $A = \{1, 2, 3, 4, 5, 6\}$

(i) $5 \in A$ (ii) $8 \notin A$ (iii) $0 \notin A$ (iv) $4 \in A$

(v) $2 \in A$ (vi) $10 \notin A$

3.

(i) $A = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

(ii) $B = \{1, 2, 3, 4, 5\}$

(iii) $C = \{17, 26, 35, 44, 53, 62, 71, 80\}$

(iv) $D = \{2, 3, 5\}$

(v) $E = \{T, R, I, G, O, N, M, E, Y\}$

(vi) $F = \{B, E, T, R\}$

4.

(i) $A = \{x : x = 3n, n \geq 4, n \in N\}$

(ii) $B = \{x : x = 2^n, n \geq 5, n \in N\}$

(iii) $C = \{x : x = 5^n, n \geq 4, n \in N\}$

(iv) $D = \{x : x = 2n, n \in N\}$

(v) $E = \{x : x = n^2, n \geq 10, n \in N\}$

5

(i) $A = \{1, 3, 5, \ldots\}$

(ii) $B = \{0, 1, 2, 3, 4\}$

(iii) $C = \{0, \pm 1, \pm 2\}$

(iv) $D = \{L, O, Y, A\}$

(v) $E = \{$ February, April, June, September, November $\}$

(vi) $F = \{b, c, d, f, g, h, j\}$

6.

(i) – (c)

(ii) – (a)

(iii) – (d)

(iv) – (b)

$-\frac{1}{2} = -0.$

$\frac{9}{2} = 4.$

EXAM

1\. (a) Yes

They are a well defined collection of distinct objects.

(b) No.

Determining the tallness of a person vary.

2\. $x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$(x-1)(x+2) = 0$

$x = -2, 1$

$A = [-2, 1]$

3. (d) $8 \in A$

4. $A = \left[\frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}, \frac{5}{8}, \frac{6}{9}, \frac{7}{10}\right]$

5. $B = [0, \pm 1, \pm 2, \pm 3, \pm 4]$

6. $C = \{x : x = \frac{n}{5}, n \in N\}$

7. $D = \{x : x = \frac{n}{n^2+1}, n \in N\}$

CARDINAL NUMBER [CARDINALITY]

No. of elements in a set is called cardinality.

* No. of elements in set A is generally denoted by ~~n(A)~~. n(A)

Eg: $A = \{5, 9, 11\}$

$n(A) = 3$

: $B = \{a, e, i, o, u\}$

$n(B) = 5$

* Cardinality is defined only for finite set.

③ <u>Finite Set</u>

Finite sets are those having <u>finite no. of elements</u>

Eg: x = $\{5, 6, 7, 9\}$

: A set of even natural numbers less than hundred.

④ <u>Infinite Set</u>

Infinite sets are those having <u>infinite no. of elements.</u>

Eg: Set of all natural numbers

$N = \{1, 2, 3, \ldots\}$

: Set of all real numbers

: Set of all points in a plane.

⑤ <u>Equivalent Set</u>

Equivalent sets are those having <u>same number of elements.</u>

Eg: $A = \{1, 2, 3, 4, 5\} \Rightarrow n(A) = 5$

$B = \{a, e, i, o, u\} \Rightarrow n(B) = 5$

Here $n(A) = n(B)$

$\therefore$ Equivalent.

⑥ <u>Equal Set</u>

Two sets A & B are said to be equal if they have exactly same elements and we write <u>$A = B$</u>. Otherwise, the sets are said to be unequal and we write $A \neq B$

TYPES OF SETS

① Empty [Void / Null] Set

A set which does not contain any element is called empty set.

* Empty set is generally denoted by using ϕ (read as 'phi') or { }
* Cardinality is zero for a null set.

Eg: $A = \{x : 1 < x < 2, x \in N\}$

$= \{\ \} = \phi$

Here $n(A) = 0$

: $B = \{x : x^2 = 4, x \text{ is odd}\}$

$= \phi$

$C = \{x : x$ is a point common to any two parallel lines$\}$

② Singleton set

A set is ~~soted~~ said to be a singleton set if it contains only one element

* Cardinality is one for a singleton set

Eg: $A = \{x : 1 < x < 3, x \in N\}$

$= \{2\}$

Here $n(A) = 1$

~~: $B = \{x : x = 7, x \in N\}$~~

: $B = \{x : |x| = 7, x \in N\}$

Eg. $A = \{x : x$ is a letter of the word FATHER$\}$

$= \{F, A, T, H, E, R\}$

$B = \{x : x$ is a letter of the word FEATHER$\}$

$= \{F, E, A, T, H, R\}$

Here $A = B$

$\therefore A = \{1, 2, 3, 4\}$

$B = \{3, 4, 2, 1\}$

Here $A = B$

Q. State which of the following sets are equivalent or not.

1) $A = \{x / x$ is a day of a week$\}$

$B = \{1, 2, 3, 4, 5, 6, 7\}$

Solution

$A = \{$Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday$\} \Rightarrow n(A) = 7$

$B = \{1, 2, 3, 4, 5, 6, 7\} \Rightarrow n(A) = 7$

Here $n(A) = n(B)$

Hence A & B are equivalent.

2) $A = \{a, e, i, o, u\}$

$B = \{5, 10, 15, 20, 25\}$

Solution

$n(A) = n(B) = 5$

Here A & B are equivalent.

$\therefore A = B$

3) $A = \{1, 2, 3\}$

$B = \{x : x$ is a solution of the equation $(x-1)(x-2)(x-3) = 0\}$

Solution

$(x-1)(x-2)(x-3) = 0$

$x - 1 = 0 \quad x - 2 = 0 \quad x - 3 = 0$

$x = 1 \quad x = 2 \quad x = 3$

$B = \{1, 2, 3\}$

Hence $A = B$

4) $A = \{x : x$ is a day in a week$\}$

$B = \{x : x$ is a vowel$\}$

Solution

$A = \{$Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday$\}$

$B = \{a, e, i, o, u\}$

Hence $A \neq B$

SUBSET OF A SET

A set A is said to be a subset of a set B, if every element of A is also an element of B and we write $A \subset B$

* If $A \subset B$, then B is called superset of A

i.e, $A \subset B$ if $a \in A \Rightarrow a \in B$

* If A is not a subset of B, we write it as $A \not\subset B$

Eg: $A = \{1, 2, 5\}$ & $B = \{1, 2, 3, 4, 5\}$

$A \subset B$

3) $A = \{x / x$ is a higher secondary school in Kerala$\}$

$B = \{x / x$ is a district of Kerala$\}$

Solution

$n(B) = 14$

$n(A) \neq n(B)$

Hence A & B are not equivalent.

Q7 State which of the following sets are equal or not

1) $A = \{x / x \in N$ and $x - 5 = 0\}$

$B = \{x / x \in N$ and $x^2 - 25 = 0\}$

Solution

$x - 5 = 0$

$\Rightarrow x = 5$

$\therefore A = \{5\}$

$x^2 - 25 = 0$

$\Rightarrow x^2 = 25$

$x = \pm 5$

$x = 5 \quad (\because x \in N)$

$\therefore B = \{5\}$

$\therefore A = B$

2) $A = \{x / x$ is a letter of the word LOYAL$\}$

$B = \{x / x$ is a letter of the word ALLOY$\}$

Solution

$A = \{L, O, Y, A\}$

$B = \{A, L, O, Y\}$

Eg: (i) $2 \leq x \leq 5 \Rightarrow x \in [2,5]$

(ii) $2 < x < 5 \Rightarrow x \in (2,5)$

(iii) $2 \leq x < 5 \Rightarrow x \in [2,5)$

(iv) $2 < x \leq 5 \Rightarrow x \in (2,5]$

NOTES

(i) $x \geq a \Rightarrow x \in [a, \infty)$

$\downarrow$

closed interval a to infinity

(ii) $x > a \Rightarrow x \in (a, \infty)$

$\downarrow$

open interval a to infinity

(iii) $x \leq a \Rightarrow x \in (-\infty, a]$

$\downarrow$

open interval -ve infinity to closed interval a

(iv) $x < a \Rightarrow x \in (-\infty, a)$

$\downarrow$

Open interval -ve infinity to open interval a

Eg: (i) $x \geq 2 \Rightarrow x \in [2, \infty]$

(ii) $x > 2 \Rightarrow x \in (2, \infty)$

(iii) $x \leq 2 \Rightarrow x \in (-\infty, 2]$

(iv) $x < 2 \Rightarrow x \in (-\infty, 2)$

$\infty \rightarrow$ very large +ve number
$-\infty \rightarrow$ very large -ve number

NOTES

(i) Every set is subset of itself i.e., $A \subset A$

(ii) ϕ is a subset of all sets i.e., $\phi \subset A$

(iii) If $A \subset B$ and $B \subset A$ then $A = B$

(iv) Number system

$$\boxed{N \subset W \subset Z \subset Q \subset R \subset C}$$

$$\boxed{T \subset R \subset C}$$

Complex numbers is the superset of all number system

INTERVALS AS A SUBSET OF ALL REAL NUMBERS

(i) $\boxed{a \le x \le b \Rightarrow x \in [a, b]}$

Closed interval a to b

(ii) $\boxed{a < x < b \Rightarrow x \in (a, b)}$

Open interval a to b

(iii) $\boxed{a \le x < b \Rightarrow x \in [a, b)}$

Closed interval a to open interval b

(iv) $\boxed{a < x \le b \Rightarrow x \in (a, b]}$

Open interval a to closed interval b

PROPER SUBSET

Set A is said to be a proper subset of set B, if A is a subset of B and $A \neq B$ Then we write $A \subset B$

* $A \subseteq B$: A is subset or equal to B
* If A is a proper subset of B, then B must have atleast one element which is not in A.

Eg: $A = \{a, b, c\}$

$B = \{a, b, c, \boxed{d, e}\}$

$A \subset B$

There are two elements in B which are not A. Hence A is a proper subset of B.

POWER SET

Set of all possible subsets of a set is called powerset

* Power set of A is generally denoted by P(A)

Eg: $A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

: $B = \{5, 7\}$

$P(B) = \{\phi, \{5\}, \{7\}, \{5, 7\}\}$

: $C = \{9\}$

$P(C) = \{\phi, C\}$

NOTES [IMP]

If $n(A) = n$ then,

(i) $n[P(A)] = 2^n$

(ii) Total number of possible subsets of $A = 2^n$

(iii) Total number of non-empty possible subsets of $A = 2^n - 1$

(iv) Total number of proper subsets possible for $A = 2^n - 1$

(v) Total number of non-empty proper subsets possible for $A = 2^n - 2$

DIFFERENCE BETWEEN $\in$ AND $\subset$

Eg: $A = \{1, 2, 3, 4\}$

$1 \in A$	$\{1\} \subset A$
$2 \in A$	$\{2\} \subset A$
$3 \in A$	$\{3\} \subset A$
$4 \in A$	$\{4\} \subset A$
	$\phi \subset A$

: $B = \{1, 2, \{3\}, 4\}$

$1 \in B$	$\{1\} \subset B$
$2 \in B$	$\{2\} \subset B$
$\{3\} \in B$	$\{\{3\}\} \subset B$
$4 \in B$	$\{4\} \subset B$
	$\{1, 2\} \subset B$
	$\{1, \{3\}\} \subset B$
	$\{1, 4\} \subset B$
	etc.

$\phi \subset B$

$D = \{1, \{2,3\}, 4\}$

$1 \in D$ $\quad$ $\{1\} \subset D$

$\{2,3\} \in D$ $\quad$ $\{\{2,3\}\} \subset D$

$4 \in D$ $\quad$ $\{4\} \subset D$

$\phi \subset D$

$E = \{\phi, 1, 2, 3\}$

$\phi \in F$ $\quad$ $\{\phi\} \subset F$

$1 \in F$ $\quad$ $\{1\} \subset F$

$2 \in F$ $\quad$ $\{2\} \subset F$

$3 \in F$ $\quad$ $\{3\} \subset F$

$\phi \subset F$

i.e., Here $\phi \in F$, $\phi \subset F$, $\{\phi\} \subset F$

UNIVERSAL SET

The super set of all the sets under consideration is known as universal set.

* Generally denoted by **U**

Eg: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 4, 6, 8\}$

$B = \{1, 3, 5, 7, 9\}$

Here, $A \subset U$, $B \subset U$, $C \subset U$

i.e., U is the super set of all given set Hence U is called the universal set

NOTES

In number system.

$\boxed{N \subset W \subset Z \subset Q \subset R \subset C}$

$\boxed{T \subset R \subset C}$

Here complex number is a super set of all number system. Hence complex number is the universal set of all number system

$R^+ \rightarrow$ +ve real number

$R^- \rightarrow$ -ve real number

$Q^+ \rightarrow$ +ve rational number

$Q^- \rightarrow$ -ve rational number

Q3 Consider the following sets.

$A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5\}$ $C = \{1, 2, 3, 4, 5, 6, 7\}$

Insert the symbol $\subset$ / $\not\subset$ between each of the following pair of sets.

(i) A $\subseteq$ B (ii) B $\not\subseteq$ A (iii) A $\subseteq$ C (iv) B $\subseteq$ C

(v) C $\not\subset$ B (vi) C $\not\subseteq$ A

Q4. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are true.

(i) $\{3, 4\} \subset A$ F

(ii) $\{3, 4\} \in A$ T

(iii) $\{\{3, 4\}\} \subset A$ T

(iv) $1 \in A$ T

(v) $1 \subset A$ F

(vi) $\{1, 2, 5\} \subset A$ T

(vii) $\{1, 2, 5\} \in A$ F

(viii) $\{1, 2, 3\} \subset A$ F

(ix) $\emptyset \in A$ F

(x) $\emptyset \subset A$ T

(xi) $\{\emptyset\} \subset A$ F

Q10 A set contains n elements, its power set contains

(a) n elements (b) 2^n elements

(c) n^2 elements (d) None of these.

Solution

$n(A) = n$

$n[P(A)] = 2^n$

(b) 2^n elements.

Q11 Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

Solution

$n(A) = m$

$n[P(A)] = 2^m$

$n(B) = n$

$n[P(B)] = 2^n$

$2^m = 2^n + 56$

$2^m - 2^n = 56$ $\qquad 2^6 - 2^3 = 56$

$m = 6$

$n = 3$

$2^2 = 4$

$\boxed{2^3 = 8}$

$2^4 = 16$

$2^5 = 32$

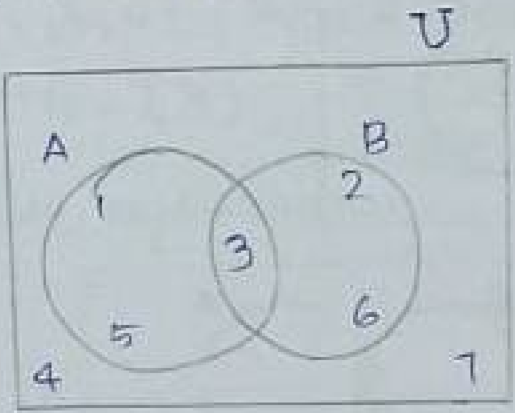
$\boxed{2^6 = 64}$

VENN DIAGRAM

It is a diagramatic representation of sets in such a way that universal set is represented by rectangle and all other sets by circle which lies inside of the rectangle.

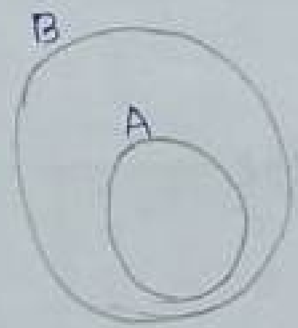
eg: $U = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 3, 5\}$ $\quad B\ \{2, 3, 6\}$



NOTES

If $A \subset B$ then



OPERATION ON SETS

I Union Of Sets ($\cup$)

The union of two sets A and B, denoted by $A \cup B$ is the set of all those elements, each one of which is either in A or in B or in both A and B

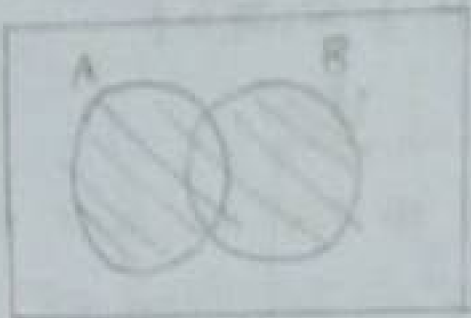
$$\boxed{A \cup B = \{x : x \in A / x \in B]}$$

* If $x \in A \cup B$ then $x \in A$ or $x \in B$
* If $x \notin A \cup B$ then $x \notin A$ and $x \notin B$

Eg: $A = \{1, 3, 5\}$, $B = \{2, 3, 6\}$

$A \cup B = \{1, 2, 3, 5, 6\}$

* Venn diagram representation of $A \cup B$



Properties of union

(1) Commutative property.

$$A \cup B = B \cup A$$

(2) Associative property

$$A \cup (B \cup C) = (A \cup B) \cup C$$

(3)

$$A \subset (A \cup B)$$
$$B \subset (A \cup B)$$

(4) Existence of identity element

$$A \cup \phi = A = \phi \cup A$$

(5) Idempotent law

$$A \cup A = A$$

(6) Law of U

$$A \cup U = U$$

11 Intersection of Sets ($\cap$)

The intersection of two sets A and B, denoted by $A \cap B$ is the set of all elements, common to both A and B

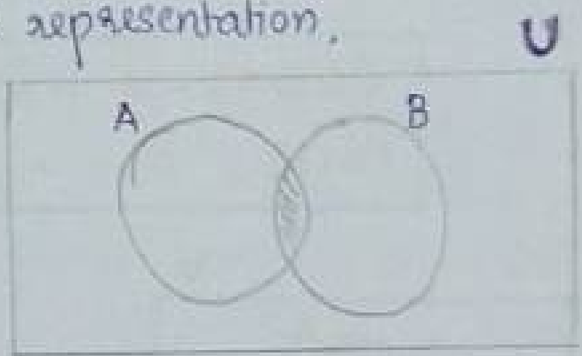
$$\boxed{A \cap B = \{x : x \in A \text{ and } x \in B\}}$$

* If $x \in A \cap B$, then $x \in A$ and $x \in B$
* If $x \in A \cap B$, then $x \notin A$ or $x \notin B$

Eg: $A = \{1, 3, 5\}$ $B = \{2, 3, 6\}$

$A \cap B = \{3\}$

* Venn diagram representation.

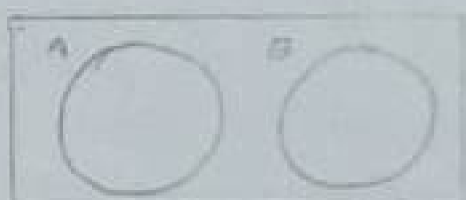


NOTES

• Disjoint sets

$A \cap B = \phi$

$n(A \cap B) = 0$



Eg: $C = \{2, 4, 6\}$ $D = \{1, 3, 5\}$

$C \cap D = \phi$

Hence C & D are disjoint.

Properties of Intersection.

(1) Commutative Property

$$A \cap B = B \cap A$$

(2) Associative property.

$$A \cap (B \cap C) = (A \cap B) \cap C$$

(3) ~~Distributive~~ Distributive property.

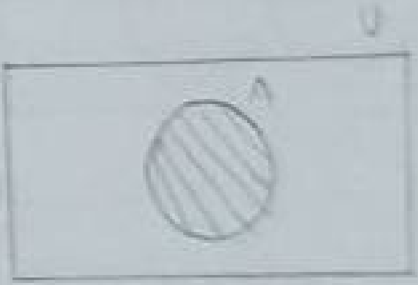
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(4) Idempotent law

$$A \cap A = A$$

(5) $A \cap U = A$



(6) $A \cap \phi = \phi$

III Difference of Sets

If A and B are two sets, then their difference A − B is the set of all those elements of A which do not belong to B

$$\boxed{A - B = \{x : x \in A \text{ and } x \notin B\}}$$

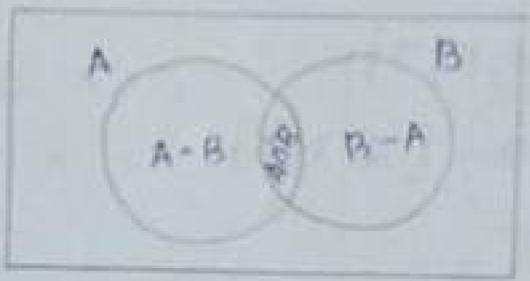
$$B - A = \{x : x \in B \text{ and } x \notin A\}$$

Eg. $A = \{1,3,5\}$ $B = \{2,3,6\}$

$A - B = \{1,5\}$

$B - A = \{2,6\}$

* Venn Diagram Representation



$$\boxed{\begin{aligned} A - B &= A - (A \cap B) \\ B - A &= B - (B \cap A) \end{aligned}}$$

Properties

(1) $(A-B) \cup (B-A) \cup (A \cap B) = A \cup B$

(2) $(A-B) \cup (A \cap B) = A$

$(B-A) \cup (A \cap B) = B$

(3) $(A-B) \cup B = A \cup B$

$(B-A) \cup A = A \cup B$

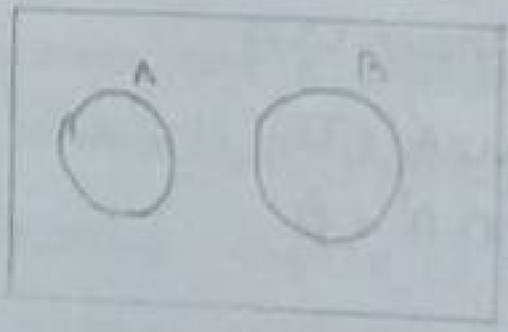
(4) Disjoint Sets

$A - B = A - (A \cap B)$

$A - B = A$

Similarly,

$B - A = B$



IV Compliment of A Set

* Compliment of set A is generally denoted by A', A^c or $\bar{A}$ and defined as $A' = U - A$.
* ~~A - B = {x : x ∈ A and x ∉ B}~~ $A' = \{x, x \in U \text{ and } x \notin A$
* $x \in A' \Leftrightarrow x \notin A$.

Eg: $U = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 3, 5\}$ $B = \{2, 3, 6\}$

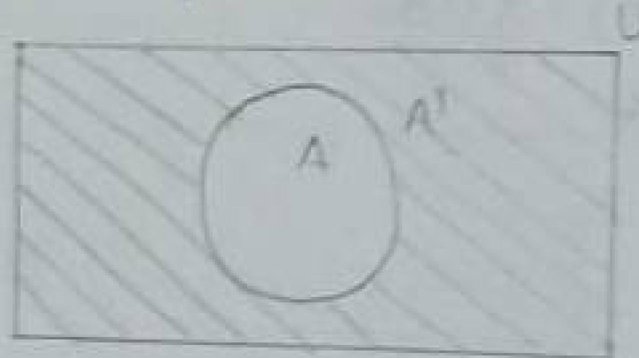
$A' = U - A$

$= \{2, 4, 6, 7\}$

$B' = U - B$

$= \{1, 4, 5, 7\}$

* Venn diagram representation



Properties

(1) Compliment Laws

$A \cup A' = U$

$A \cap A' = \phi$

(2) De-morgan's Law

$$(A\cup B)' = \cancel{A'\cup}\ A'\cap B'$$

$$(A\cap B)' = A'\cup B'$$

(3) $(A')' = A$

<u>Proof</u>

$$(A')' = U - A'$$
$$= A.$$

(4) * $\phi' = U - \phi$

$= U$

$\phi' = U$

* $U' = U - U$

$U' = \phi$

Q12 Find the union of each of the following pair of sets.

1) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

2) $A = \{1, 3, 5\}$ $B = \{1, 2, 3\}$

3) $A = \{x : x$ is a natural number and a multiple of $3\}$

$B = \{x : x$ is a natural number less than $6\}$

4) $A = \{x : x$ is a natural number and $1 < x < 6\}$

$B = \{x : x$ is a natural number and $6 < x < 10\}$

5) $A = \{1, 2, 3\}$ $B = \phi$

Solution

1) $A \cup B = \{a, e, i, o, u\} \cup \{a, b, c\}$

$= \{a, b, c, e, i, o, u\}$

2) $A \cup B = \{1, 3, 5\} \cup \{1, 2, 3\}$

$= \{1, 2, 3, 5\}$

3) $A = \{3, 6, 9, 12, 15, \ldots\ldots\}$

$B = \{1, 2, 3, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12, 15, \ldots\ldots\}$

4) $A = \{2, 3, 4, 5\}$

$B = \{7, 8, 9\}$

$A \cup B = \{2, 3, 4, 5, 7, 8, 9\}$

5) $\cancel{A \cup \phi}$ $A \cup B = \{1, 2, 3\} \cup \phi$

$= \{1, 2, 3\}$

Q8 If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Solution

$A \cup B = B$

* If $A \subset B$ then $A \cap B = A$

Q. If $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$ $C = \{5, 6, 7, 8\}$, then verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

LHS

$B \cap C = \{5, 6\}$

$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\}$

RHS

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\}$

LHS = RHS

Thus proved.

Q. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$ then find

1) $X - Y$ 2) $Y - X$ 3) $X \cap Y$

Solution

1) $X - Y = \{a, b, c, d\} - \{f, b, d, g\}$
$= \{a, c\}$

2) $Y - X = \{f, d, b, g\} - \{a, b, c, d\}$
$= \{f, g\}$

3) $X \cap Y = \{b, d\}$

Q6. If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$?

Solution

We know that $R = Q \cup T$

$\therefore R - Q = T$, irrational numbers.

Q7. Let $U = \{1, 2, 3, 4, \ldots\ldots, 9\}$ $A = \{1, 2, 3, 4\}$
$B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$

1) A' 2) B' 3) $(A \cup C)'$ 4) $(A \cup B)'$ 5) $(A')'$ 6) $(B - C)$

Solution

1) $A' = U - A$
$= \{5, 6, 7, 8, 9\}$

2) $B' = U - B$
$= \{1, 3, 5, 7, 9\}$

3) $A \cup C = \{1, 2, 3, 4, 5, 6\}$
$(A \cup C)' = U - (A \cup C)$
$= \{7, 8, 9\}$

4) $A \cup B = \{1, 2, 3, 4, 6, 8\}$
$(A \cup B)' = U - (A \cup C)$
$= \{5, 7, 9\}$

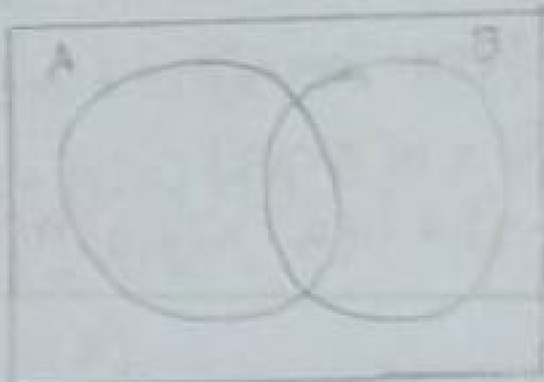
5) $(A')' = A$
$= \{1, 2, 3, 4\}$

6) $B - C = \{2, 4, 6, 8\} - \{3, 4, 5, 6\}$
$= \{2, 8\}$

$(B-C)' = U - (B-C)$
$= \{1, 3, 4, 5, 6, 7, 9\}$

PRACTICAL PROBLEMS ON SET

① $\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$



NOTE

If A & B are disjoint then $A \cap B = \phi$

$\Rightarrow n(A \cap B) = 0$

$\Rightarrow n(A \cup B) = n(A) + n(B)$

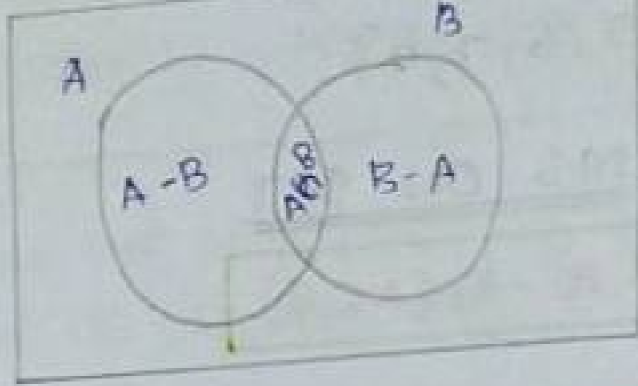
② $\boxed{n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)}$

NOTE

If A & B are disjoint then

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$

(3)



$n(A) = n(A-B) + n(A \cap B)$

$n(B) = n(B-A) + n(A \cap B)$

$n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$

Q. If X and Y are sets such that X∪Y has 50 elements, X has 23 elements and Y has 32 elements. How many elements does in X∩Y

Solution

$n(X \cup Y) = 50$

$n(X) = 23$

$n(Y) = 32$

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

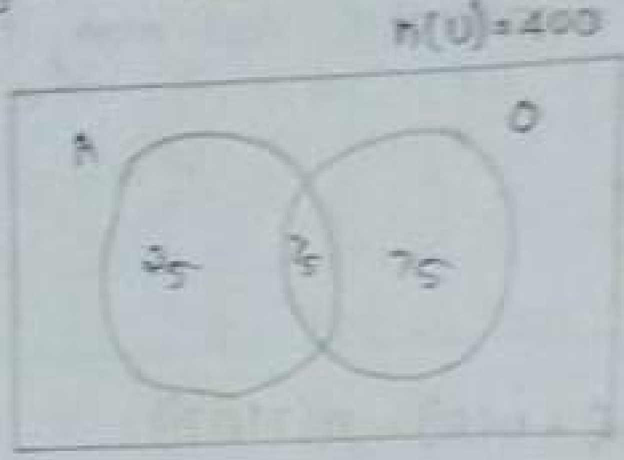
$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$

$= 23 + 32 - 50$

$= 10$

Solution

Venn diagram method.



No. of students who took neither apple nor orange juice.

$= 400 - [25 + 75 + 75]$

$= 400 - 175$

$= \underline{\underline{225}}$

$\boxed{\text{OR}}$

$n(A) = 100$

$n(O) = 150$

$n(A \cap O) = 75$

$n(U) = 400$

No. of students was taking neither apple nor orange juice $= n(A' \cap O')$

$= n[(\cancel{A \cap} A \cup O)']$

$= n(U) - n(A \cup O)$

$= 400 - 175$

$= \underline{\underline{225}}$

Q19. In a school 20 teachers ~~who~~ teaches mathematics or physics. Of these, 12 teache mathematics and 4 teach both physics and mathematics. How many teach physics?

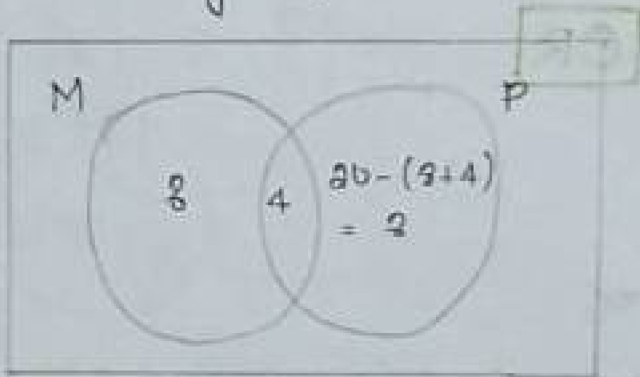
Solution

$$n(M \cup P) = 20$$
$$n(M) = 12$$
$$n(M \cap P) = 4$$

$$n(M \cup P) = n(M) + n(P) - n(M \cap P)$$
$$20 = 12 + n(P) - 4$$
$$20 = 8 + n(P)$$
$$12 = n(P)$$

$\boxed{\text{OR}}$

Using Venn diagram,



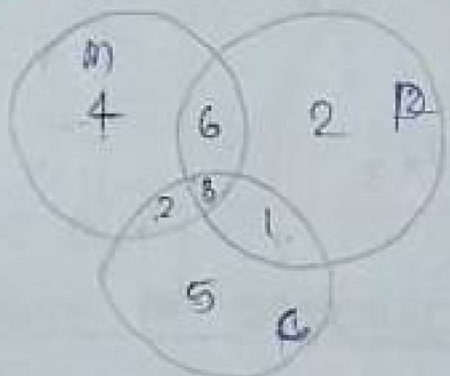
$$n(P) = 4 + 8 = 12$$

Q20. In a survey of 400 students in a school, 100 students are listed as taking apple juice, 150 taking orange juice and 75 listed as taking both apple as well as orange juice. Find how many students took neither apple nor orange juice.

Q. In a survey of 25 students, it was found that 15 had taken mathematics, 12 has taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 has taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the 3 subjects. Find the number of students that had taken.

(1) Only chemistry (2) Only mathematics (3) Only physics.
(4) Physics and mathematics but not chemistry.
(5) Mathematics and chemistry but not physics.
(6) Physics and chemistry but not mathematics.
(7) Exactly one of the subjects
(8) Exactly two of the subjects.

Solution



(1) 5 (2) 4 (3) 2 (4) 6 (5) 2 (6) 1 (7) $4+2+5=11$
(8) $6+1+2=9$.

Q 22. In a class of 60 students, 23 play hockey, 15 play basket ball and 20 play cricket, 7 play hockey and basket ball, 5 play cricket and basket ball, 4 play hockey and cricket and 15 students do not play any of these games. Find the following.

(1) How many play all the 3 games.

(2) How many play basket ball only.

(3) How many play hockey only.

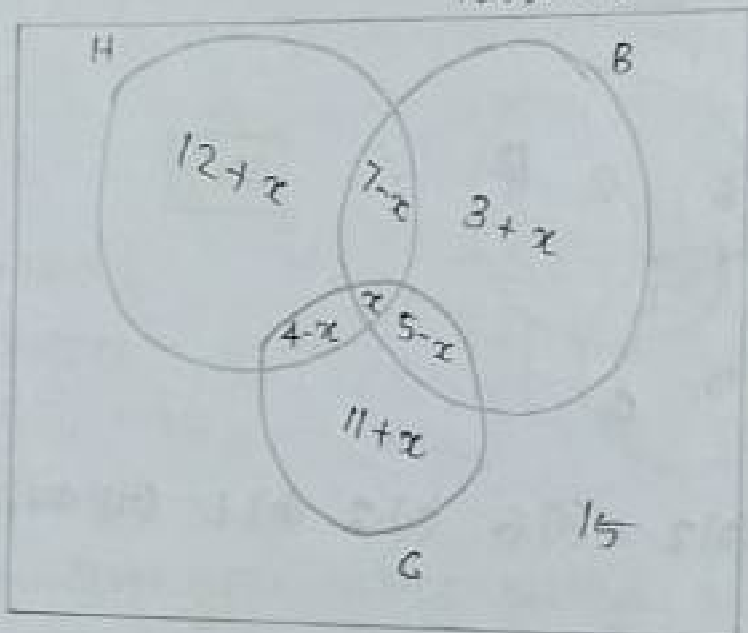
(4) How many play cricket only.

(5) How many play exactly one of the games

(6) How many play exactly two of the games

Solution

$n(U) = 60$



$x + (7-x) + (4-x) + (5-x) + (12+x) + (3+x) + (11+x) + 15 = 60$

$x + 57 = 60$

$x = 60 - 57$

$= \underline{\underline{3}}$

(1) 3 (2) 6 (3) 15 (4) 14 (5) 35 (6) 7