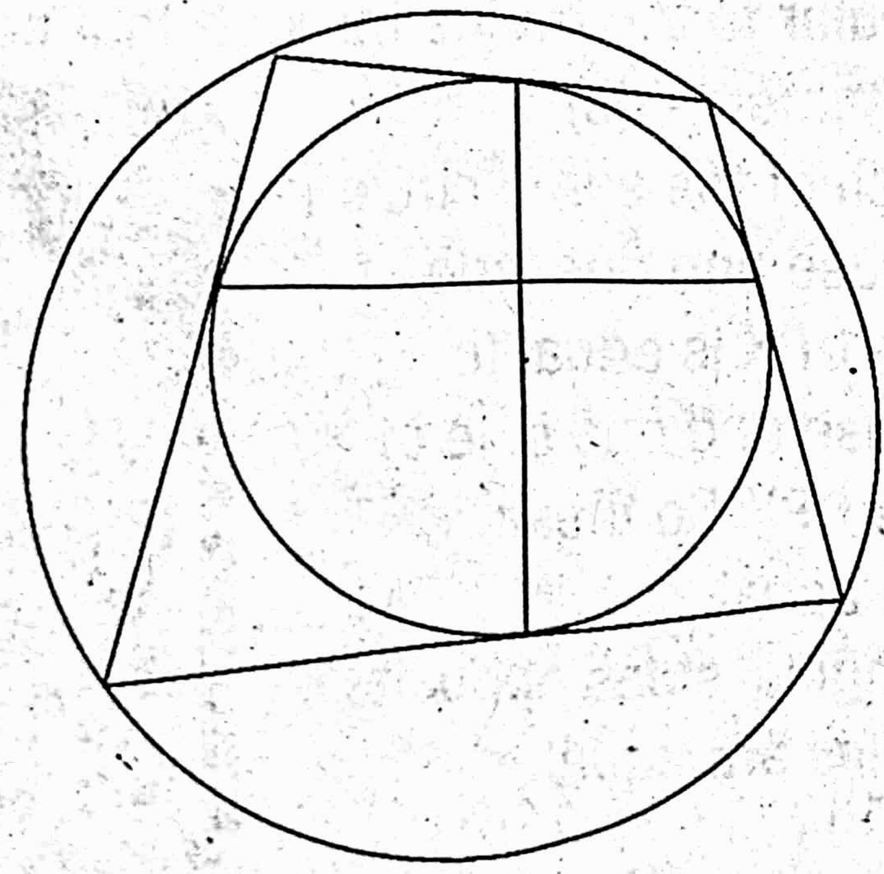
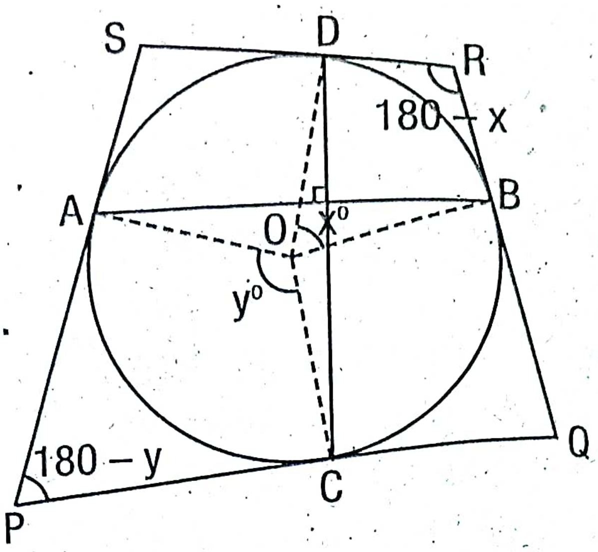
4. Prove that the quadrilateral with sides as the tangents at the ends of a pair of perpendicular chords of a circle is cyclic. (See the problem (7) in the section, chord, angle and arc of the chapter Circles.) What sort of a quadrilateral do we get if one chord is a diameter?

And if both chords are diameters?



AB and CD are two perpendicular chords of a circle with centre O. We have seen in the chapter 'Circles' that the opposite arcs formed by the end points of these chords joined together would make half the circle. That means the sum of the central angle of these arcs is 180°.



If the central angle of arc DB is x° and the central angle of arc AC is y°, then $x + y = 180°$ ..... (1)

$\angle APC = 180 - y$ ..... (2) (In a circle the angles between the radii through two points and the angles between the tangents at these points are supplementary.)

In the same way, $\angle DRB = 180 - x$ ...... (3)

(2) + (3), $\angle APC + \angle DRB = 180 - y + 180 - x$

$= 360 - x - y$

$= 360 - (x + y)$

$= 360 - 180$ (from (1))

$= 180°$

Since the opposite angles of quadrilateral PQRS are supplementary, it is cyclic.

If one chord in it is a diameter, the tangents at its ends will be parallel. Then the quadrilateral got will be a trapezium.

As it is cyclic, it will be an isosceles trapezium.

(See Textbook page 59, question 4.)

If both chords are diameters then the quadrilateral obtained willl be a square.