

3. Prove that in any square pyramid, the squares of the height, slant height and lateral edge are in arithmetic sequence.

Let 'h' be the height and 'a' be the base edge. $(\text{height})^2 = h^2$

$$(\text{Slant height})^2 = h^2 + \left(\frac{a}{2}\right)^2 = h^2 + \frac{a^2}{4}$$

$$\text{Diagonal of base} = a\sqrt{2}$$

$$\begin{aligned} (\text{Lateral edge})^2 &= h^2 + \left(\frac{a\sqrt{2}}{2}\right)^2 \\ &= h^2 + \frac{2a^2}{4} = h^2 + \frac{a^2}{4} + \frac{a^2}{4} \end{aligned}$$

Thus we get the sequence of the squares of the height, slant height and lateral edge as h^2 , $h^2 + \frac{a^2}{4}$, $h^2 + \frac{a^2}{4} + \frac{a^2}{4}$.

There is a common difference $\frac{a^2}{4}$, so it is in arithmetic sequence.

1. What is the volume of a square pyramid of base edge 10 centimetres and slant height 15 centimetres?

Height of the pyramid,

$$h = \sqrt{l^2 - \left(\frac{a}{2}\right)^2}$$

$$\left. \begin{array}{l} a = 10 \\ l = 15 \end{array} \right\}$$

$$= \sqrt{15^2 - 5^2} = \sqrt{225 - 25}$$

$$= \sqrt{200} = \sqrt{100 \times 2}$$

$$= 10\sqrt{2} \text{ cm}$$

Volume of the square pyramid

$$= \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times 10 \times 10 \times 10\sqrt{2}$$

$$= \frac{1000\sqrt{2}}{3} \text{ cu.cm.}$$