Accuracy, precision of instruments and errors in measurement

The difference between the measured value and the true value of physical quantity is called the error in its measurement. Error in measurement may arise due to several sources, and can be broadly classified into two categories.

1. **Systematic errors** - due to known causes.
   Some important causes for this are:
   (a) Limitations of the calibration of the instrument.
   (b) Limitations of the method used for measurement.
   (c) Personal errors
2. **Random errors** - due to unknown causes.
   It depends on the qualities of the measuring person and the care taken in the measuring process.

Least count error: It is an error associated with the resolution (the least value that can be measured) of the instrument. This can be reduced) by using high precision instrument, repeating the experiment and find the mean value.

2.6.1. Absolute error

The difference in the magnitudes of the true value and the measured value of a physical quantity is called absolute error.

If a_1, a_2, -----, a_n are the measured values of a certain quantity, the errors $\Delta a_1, \Delta a_2 \ldots \Delta a_n$ in the measurements are

$$\Delta a_1 = a_1 - a_{mean}$$

$$\Delta a_2 = a_2 - a_{mean} \quad ; \Delta a_3 = a_3 - a_{mean}$$

$$\text{where } a_{mean} = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$$

Absolute error $|\Delta a|$ will always be positive.

(b) Mean absolute error: The arithmetic mean of all absolute errors in the measured value is called mean absolute error.

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + ----- + |\Delta a_n|}{n}$$

The value obtained in a single measurement may be in the range $a_{mean} \pm \Delta a_{mean}$.

(c) **Relative error and percentage error:** The ratio of mean absolute error to the mean value of the quantity being measured is called relative error or fractional error. Relative error = $\frac{\Delta a_{mean}}{a_{mean}}$

Percentage error = Relative error × 100

$$= \frac{\Delta a_{mean}}{a_{mean}} \times 100$$

Combination of errors

1. **Error of a sum, or a difference**

   Let two quantities A and B have measured values $A \pm \Delta A$ and $B \pm \Delta B$ respectively.
   The maximum possible errors in the sum or difference is given by $\Delta Z = \Delta A + \Delta B$.

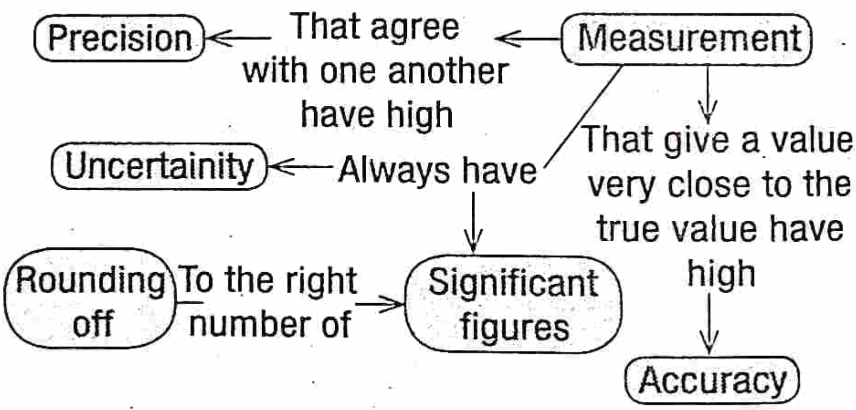
2. **Error of a product, or a quotient**

   Let two quantities A and B have measured values $A \pm \Delta A$ and $B \pm \Delta B$ respectively. When two quantities are multiplied or divided, the fractional errors in the result $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

Note: If $S = \frac{P^x Q^y}{R^z}$, then the maximum percentage error in the result is given by $\frac{\Delta S}{S} \times 100$

$$= x \times \frac{\Delta P}{P} \times 100 + y \times \frac{\Delta Q}{Q} \times 100 + z \times \frac{\Delta R}{R} \times 100$$

2.6.3. Errors in Measurement



Error in measurement is the difference between the true value and measured value of the quantity.
ie Error = True value – measured value.
Example: Suppose the actual length of an object = 3.2 cm

Measured value = 3.15 cm

Then Error = 3.2 – 3.15 = 0.05 cm (+Δx)

Here the error is positive

Suppose the measured value = 3.25 cm

Then error = 3.2 – 3.25 = –0.05 cm (–Δx)

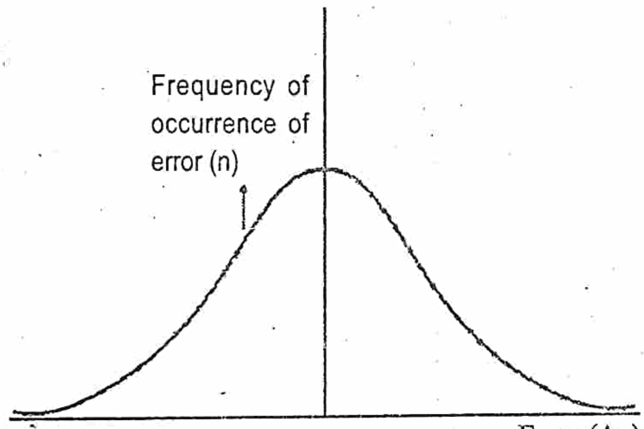
Now the error is negative.

These errors are usually assumed to follow the well known 'Gaussian law of Normal Distribution'.

Important conclusions of Gaussian law of normal distribution.

1. The probability of (+Δx) error is the same as the probability of (–Δx) error.
2. Small magnitudes of error are more than larger magnitudes of error.
3. The arithmetic mean of a large number of observations is much closer to its true value.

Note: Hence we take large number of observations and take the mean.



The 'normal distribution law' graph

The 'least counts', of the measuring instruments, used in an experiment, play a very significant role in the 'precision' associated with that experiment. Scientists are, therefore, constantly striving to design instruments, and measuring techniques, that have better (smaller) values for their 'least count'!

When we use a meter scale, we can rely on this measurement only up to a 'mm'. This is because the least count of a meter scale is 1mm only. However, a (simple) vernier caliper, can measure $\left(\frac{1}{10}\right)$th of a mm or 0.1 mm while the use of (usual) 'screw gauge' would measure up to $\left(\frac{1}{100}\right)$th of a mm or 0.01 mm.

Most of our experiments require us to use the measured values, of a number of different physical quantities, and put them in the appropriate 'formula', to calculate the required quantity. We then calculate the 'percentage reliability', or 'maximum error', in our final result:

1. By associating a 'relative error' - equal to the ratio of the least count (of the measuring instrument used) to the measured value, with each of the quantities involved in our formula.
2. By using the standard 'rules' for finding the error in a 'sum or difference', 'product or quotient', or 'power', of different quantities, involved in a given formula.

We illustrate these ideas - for calculating the maximum error - through a few examples.

Example 1: Suppose we use a physical balance to measure the mass of an object and find the mean value of our observations to be 156.28g.

Since we are somewhat uncertain, about the measurement, due to our instrument's imperfections, we need to express this in our result. Let the least count of physical balance be 0.1g. This implies that the uncertainty of any measurement made with this instrument, is ± 0.1g. Therefore, we would report the mass of this object to be (156.3g ± 0.1 g). This implies that we can only say that the mass of the object is somewhere between 156.2g and 156.4g.

Example 2: It is required to find the volume of a rectangular block. A vernier caliper is used to measure the length, width and height of the block. The measured values are found to be 1.37cm, 4.11cm and 2.56cm respectively.

Soln: The measured (nominal) volume of the block is, therefore,

$$V = l \times w \times h$$

$$= (1.37 \times 4.11 \times 2.56)\ cm^3 = 14.41\ cm^3$$

However, each of these measurements has an uncertainty of ± 0.01 cm, the least count of the vernier caliper. We can say that the values of length, width and height should be written as

$$l = (1.37\ cm \pm 0.01\ cm)$$

$$w = (4.11\ cm \pm 0.01\ cm)$$

$$h = (2.56\ cm \pm 0.01\ cm)$$

We thus find that the lower limit, of the volume of the block is given by

$$V_{min} = 1.36\ cm \times 4.10\ cm \times 2.55\ cm = 14.22\ cm^3$$

This is 0.19 cm³ lower than the (nominal) measured value.

The upper limit can also be calculated

$V_{max} = 1.38\ cm \times 4.12\ cm \times 2.57\ cm = 14.61\ cm^3$

This is 0.20 cm³ higher than the measured value.

As a practical rule, we choose the higher of these two deviations (from the measured value) as the uncertainty, in our result. We, therefore, should report the volume of the block as (14.41 cm³ ± 0.20 cm³).

Example 3: In an experiment, on determining the density of a rectangular block, the dimensions of the block are measured. with a vernier caliper (with a least count of 0.01 cm) and its mass is measured with a beam balance of least count 0.1g. How do we report our result for the density of the block?

Assume the measured values be:

Mass of block (m) = 39.3g

Length of block (l) =5.12 cm

Breadth of block (b) = 2.56 cm

Thickness of block (t) = 0.37 cm

Soln: The density of the block is given by

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{l \times b \times t}$$

$$= \frac{39.3\,g}{5.12\,cm \times 2.56\,cm \times 0.37\,cm} = 8.1037\ g\ cm^3$$

Now uncertainty in m = ± 0.1 g

uncertainty in l = ± 0.01 cm

uncertainty in b = ± 0.01 cm

uncertainty in t = ± 0.01 cm

Maximum relative error, in the density value is, therefore, given by

$$\frac{\Delta\rho}{\rho} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t} + \frac{\Delta m}{m}$$

$$= \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37} + \frac{0.1}{39.3}$$

$$= 0.0019 + 0.0039 + 0.027 + 0.0025 = 0.0353$$

Hence $\Delta\rho = 0.0358 \times 8.1037 g\ cm^{-3}$

$\cong 0.3 g\ cm^{-3}$

We cannot, therefore, report the calculated value of ρ (= 8.1037 gm⁻³) up to the fourth decimal place. Since $\Delta\rho = 0.3 g\ cm^{-3}$ the value of ρ can be regarded as accurate up to the first decimal place only. Hence the value of ρ must be rounded off as 8.1 g cm^{-3} and the result of measurements should be reported as

$$\rho = (8.1 \pm 0.3) \text{ g cm}^{-3}.$$

A careful look, at the calculations done above the main contribution to this (large) error in the measurement of ρ, is contributed by the (large) relative error (0.027) in the measurement of t, the smallest of the quantities measured. Hence the precision of the reported value of ρ could be increased by measuring 't' with an instrument having a least count smaller than 0.01 cm. Thus if a micrometer screw gauge (least count = 0.001 cm), rather than a vernier caliper were to be used, for measuring t, we would be reporting our result for ρ with a considerably lower degree of uncertainty. Experimentalists keep such facts in mind while designing their 'plan' for carrying out different measurements in a given experiment.