Qn. 1

The base radius of a cylindrical block of wood is 15 centimetres and its height is 40 cm. What is the volume of the largest cone that can be curved out of from this?

Radius of the cone $= 15$ cm

$$\text{height} = 40 \text{ cm}$$

$$\begin{aligned}
\text{Volume of cone} &= \frac{1}{3} \times \pi \times 15 \times 15 \times 40 \\
&= 3000\,\pi = 9420 \text{ cm}^3
\end{aligned}$$

Qn. 2

A solid metal cylinder is of base radius 12 cm and height 20 cm. By melting and recasting, how many cones of base radius 4 cm and height 5 cm can be made?

$$\begin{aligned}
\text{Volume of cylinder} &= \pi \times 12 \times 12 \times 20 \\
&= 2880\pi \text{ cu.cm}
\end{aligned}$$

$$\begin{aligned}
\text{Volume of one cone} &= \frac{1}{3}\pi \times 4 \times 4 \times 5 \\
&= \frac{80\pi}{3} \text{ cu.cm}
\end{aligned}$$

$$\begin{aligned}
\text{Number of cones} &= 2880\pi \div \frac{80\pi}{3} \\
&= \frac{2880\pi \times 3}{80\pi} = 108
\end{aligned}$$

Qn. 3

A sector of central angle $216°$ is cut out from a circle of radius 25 cm and it is rolled up into a cone. What is the base radius and height of the cone? What is its volume?

Slant height of the cone is equal to radius of the sector

$$\begin{aligned}
\text{Radius of the cone} &= 25 \times \frac{216}{360} \\
&= 15 \text{ cm}
\end{aligned}$$

$$\begin{aligned}
\text{Height of the cone} &= \sqrt{25^2 - 15^2} \\
&= \sqrt{625 - 225} \\
&= \sqrt{400} = 20 \text{ cm}
\end{aligned}$$

$$\begin{aligned}
\text{Volume of the cone} &= \frac{1}{3} \times \pi \times 15 \times 15 \times 20 \\
&= 1500\,\pi \\
&= 1500 \times 3.14 \\
&= 4710 \text{ cu.cm}
\end{aligned}$$

The ratio of the base radii of two cones is $3 : 5$ and their heights are in the ratio $2 : 3$. What is the ratio of their volumes?

Radius of the first cone be '$3r$' and height be '$2h$'. Radius of the second cone be '$5r$' and height be '$3h$'.

$$\text{Volume of first cone} = \frac{1}{3}\pi \times 3r \times 3r \times 2h$$
$$= 6\pi r^2 h$$

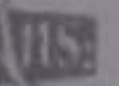$$\text{Volume of second cone} = \frac{1}{3}\pi \times 5r \times 5r \times 3h$$
$$= 25\pi r^2 h$$

$$\text{Ratio of their volume} = 6\pi r^2 h : 25\pi r^2 h$$
$$= 6 : 25$$

Qn. 5

Two cones have the same volume and their base radii are in the ratio $4 : 5$. What is the ratio of their heights?

Let radius of the first cone be '$4r$' and height be 'h_1' and radius of the second cone be '$5r$' and height be 'h_2'.

Volumes are equal.

That is $\frac{1}{3}\pi(4r)^2 \times h_1 = \frac{1}{3}\pi(5r)^2 \times h_2$

$$\frac{1}{3}\pi r^2 h_1 \times 16 = \frac{1}{3}\pi r^2 h_2 \times 25$$

$$16h_1 = 25h_2$$

$$h_1 : h_2 = 25 : 16$$