

**Q. No. 1 – 25 Carry One Mark Each**

1. Two independent random variables X and Y are uniformly distributed in the interval [-1, 1]. The probability that max [X, Y] is less than 1/2 is

- (A) 3/4                      (B) 9/16                      (C) 1/4                      (D) 2/3

Answer:- (B)

Exp:- Uniform distribution X, Y on [-1,1];  $f(x) = f(y) = \frac{1}{2}$

$$P\left(\max(x, y) \leq \frac{1}{2}\right) = P\left(X \leq \frac{1}{2}, -1 \leq Y \leq \frac{1}{2}\right) \cdot P\left(-1 \leq X \leq \frac{1}{2}, Y \leq \frac{1}{2}\right)$$

$$= \int_{-1}^{1/2} \frac{1}{2} dx \int_{-1}^{1/2} \frac{1}{2} dy = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

2. If  $x = \sqrt{-1}$ , then the value of  $x^x$  is

- (A)  $e^{-\pi/2}$                       (B)  $e^{\pi/2}$                       (C) x                      (D) 1

Answer:- (A)

Exp:- Given,  $x = \sqrt{-1}$ ;  $x^x = (\sqrt{-1})^{\sqrt{-1}} = i^i$

We know that  $e^{i\theta} = \cos \theta + i \sin \theta \Rightarrow e^{i\frac{\pi}{2}} = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} = i$

$\therefore (i)^i = (e^{i\pi/2})^i = e^{-\pi/2}$

3. Given  $f(z) = \frac{1}{z+1} - \frac{2}{z+3}$ . If C is a counter clock wise path in the z-plane such that

$|z+1| = 1$ , the value of  $\frac{1}{2\pi j} \oint_C f(z) dz$  is

- (A) -2                      (B) -1                      (C) 1                      (D) 2

Answer:- (C)

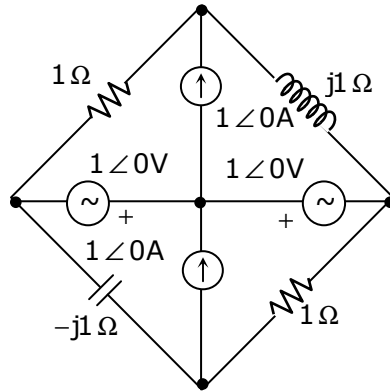
Exp:-  $\frac{1}{2\pi i} \oint_C f(z) dz = \frac{1}{2\pi i} \left[ \underbrace{\oint_C \frac{1}{z+1} dz}_{I_1} - \underbrace{\oint_C \frac{2}{z+3} dz}_{I_2} \right]$

$z = -1$  is singularity in c and  $z = -3$  is not in c

By cauchy's integral formula  $I_2 = \oint_C \frac{2}{z+3} dz = 0$

$\therefore I_1 = \oint_C \frac{1}{z+1} dz = 1$ ;  $I_1 - I_2 = 1$

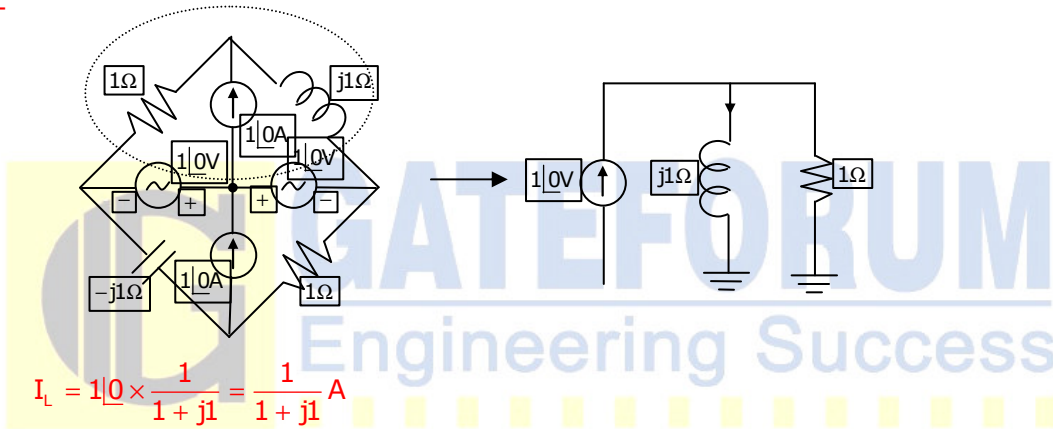
4. In the circuit shown below, the current through the inductor is



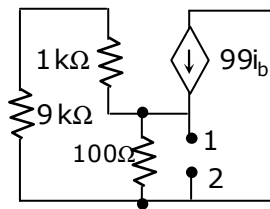
- (A)  $\frac{2}{1+j}$  A      (B)  $\frac{-1}{1+j}$  A      (C)  $\frac{1}{1+j}$  A      (D) 0 A

Answer:- (C)

Exp:-



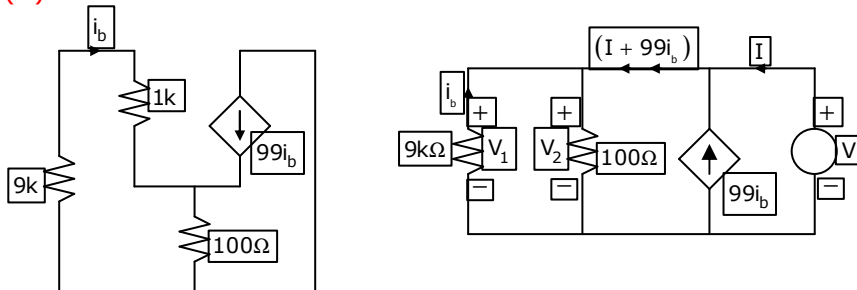
5. The impedance looking into nodes 1 and 2 in the given circuit is



- (A) 50 Ω      (B) 100 Ω      (C) 5KΩ      (D) 10.1kΩ

Answer:- (A)

Exp:-



After connecting a voltage source of V

$$V_1 = V_2 \Rightarrow (10k)(-i_b) = 100(I + 99i_b + i_b);$$

$$-10000i_b = 100I + 100 \times 100i_b = 100I + 10000i_b$$

$$-20000i_b = 100I \Rightarrow i_b = -\left(\frac{100}{20000}\right)I = \left[-\frac{I}{200}\right]$$

$$V = 100[I + 99i_b + i_b] = 100\left[I + 100\left(\frac{-I}{200}\right)\right] = 50I$$

$$R_{th} = \frac{V}{I} = \frac{50I}{I} = 50\Omega$$

6. A system with transfer function

$$G(s) = \frac{(s^2 + 9)(s + 2)}{(s + 1)(s + 3)(s + 4)}$$

is excited by  $\sin(\omega t)$ . The steady-state output of the system is zero at

- (A)  $\omega = 1\text{rad/s}$  (B)  $\omega = 2\text{rad/s}$   
(C)  $\omega = 3\text{rad/s}$  (D)  $\omega = 4\text{rad/s}$

Answer:- (C)

Exp:- Steady state output of system is

$$y(t) = |G(j\omega)| \sin(\omega t + \angle G(j\omega))$$

for y(t) to be zero

|G(jω)| can be zero

$$|G(j\omega)| = \frac{(-\omega^2 + 9)\sqrt{\omega^2 + 4}}{\sqrt{\omega^2 + 1}\sqrt{\omega^2 + 9}\sqrt{\omega^2 + 16}}$$

⇒ at  $\omega = 3 \text{ rad/sec}$

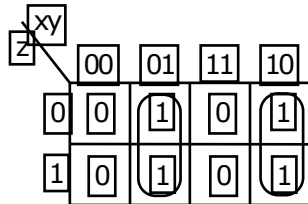
$$|G(j\omega)| = 0, \text{ thus } y(t) = 0$$

7. In the sum of product function  $f(X,Y,Z) = \sum(2,3,4,5)$ , the prime implicants are

- (A)  $\bar{X}Y, X\bar{Y}$  (B)  $\bar{X}Y, X\bar{Y}\bar{Z}, X\bar{Y}Z$   
(C)  $\bar{X}Y\bar{Z}, \bar{X}YZ, X\bar{Y}$  (D)  $\bar{X}Y\bar{Z}, \bar{X}YZ, X\bar{Y}\bar{Z}, X\bar{Y}Z$

Answer:- (A)

Exp:- .



Implicates are  $\bar{x}y\bar{z}, \bar{x}yz, x\bar{y}\bar{z}, x\bar{y}z$

The prime implicants are  $\bar{x}y$  and  $x\bar{y}$

8. If  $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$ , then the region of convergence (ROC) of its Z-transform in the Z-plane will be

- (A)  $\frac{1}{3} < |z| < 3$  (B)  $\frac{1}{3} < |z| < \frac{1}{2}$   
(C)  $\frac{1}{2} < |z| < 3$  (D)  $\frac{1}{3} < |z|$

Answer:- (C)

Exp:-  $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$

for  $(1/3)^{|n|}$  ROC is  $\frac{1}{3} < |z| < 3$

for  $(1/2)^n u[n]$  ROC is  $|z| > \frac{1}{2}$

Thus common ROC is  $\frac{1}{2} < |z| < 3$

9. The radiation pattern of an antenna in spherical co-ordinates is given by

$$F(\theta) = \cos^4 \theta; 0 \leq \theta \leq \pi/2$$

The directivity of the antenna is

- (A) 10dB (B) 12.6dB (C) 11.5dB (D) 18 dB

Answer:- (A)

Exp:- Directivity =  $\frac{|F(\theta)|_{\max}}{|F(\theta)|_{\text{ave}}}$

$$|F(\theta)|_{\max} = 1$$

$$F[\theta]_{\text{ave}} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi/2} \cos^4 \theta \sin \theta d\theta d\phi$$

$$= \frac{1}{4\pi} [2\pi] \left[ -\int_1^0 t^4 dt \right] = \frac{1}{10}$$

$$\text{Directivity} = \frac{1}{\frac{1}{10}} = 10$$

$$= 10 \log 10 = 10 \text{ dB}$$

10. A coaxial cable with an inner diameter of 1mm and outer diameter of 2.4 mm is filled with a dielectric of relative permittivity 10.89. Given

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}, \epsilon_0 = \frac{10^{-9}}{36\pi} \text{ F/m}, \text{ the characteristic impedance of the cable is}$$

- (A) 330  $\Omega$  (B) 100  $\Omega$  (C) 143.3  $\Omega$  (D) 43.4  $\Omega$

Answer:- **Answer is Not in the Options**

Exp:- Characteristic impedance

$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{2.4}{1}\right)$$

Substitute, the values, we have  $Z_0 = 15.3 \Omega$

**Note: If  $\frac{1}{2\pi}$  is not considered then the answer will be option (B)**

11. A source alphabet consists of N symbols with the probability of the first two symbols being the same. A source encoder increases the probability of the first symbol by a small amount  $\epsilon$  and decreases that of the second by  $\epsilon$ . After encoding, the entropy of the source

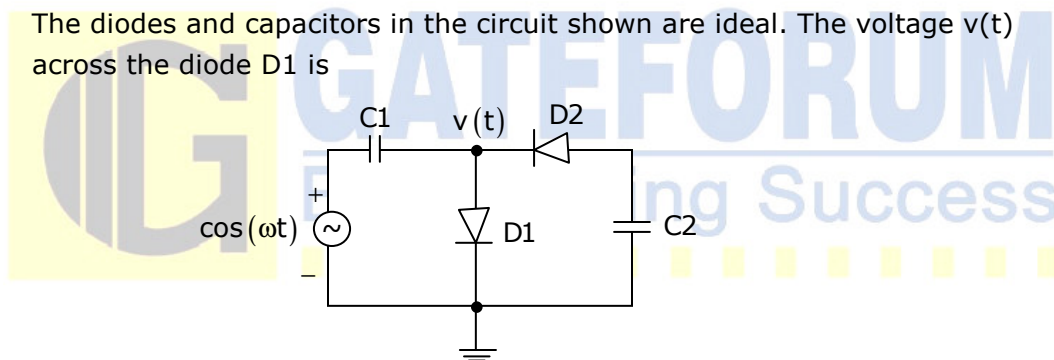
- (A) increases (B) remains the same  
(C) increases only if N = 2 (D) decreases

Answer:- (D)

Exp:- Entropy is maximum when all symbols are equiprobable

If the probability of symbols at different then entropy is going to decrease

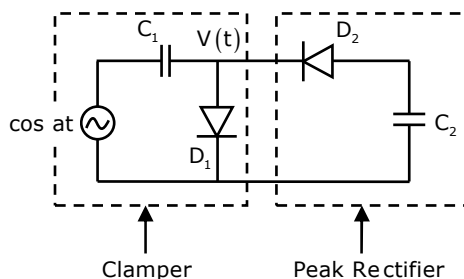
12. The diodes and capacitors in the circuit shown are ideal. The voltage  $v(t)$  across the diode D1 is



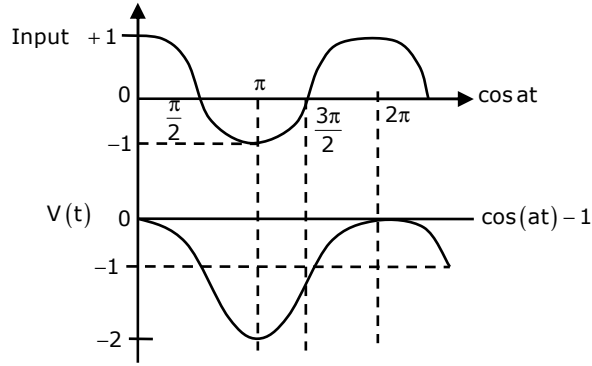
- (A)  $\cos(\omega t) - 1$  (B)  $\sin(\omega t)$   
(C)  $1 - \cos(\omega t)$  (D)  $1 - \sin(\omega t)$

Answer:- (C)

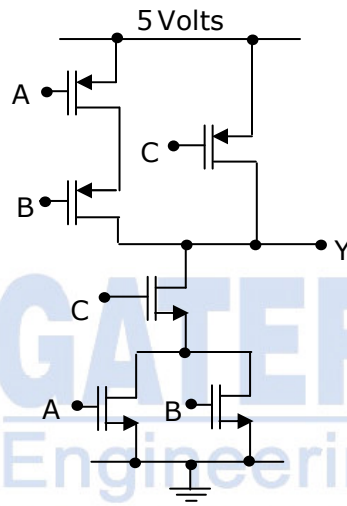
Exp:-



When excited by  $\cos(\omega t)$ , the clamping section clamp the positive peak to 0 volts and negative peak to  $-2$  volts. So whole  $\cos(\omega t)$  is lower by  $-1$  volts



13. In the circuit shown



(A)  $Y = \bar{A} \bar{B} + \bar{C}$

(B)  $Y = (A + B) \bar{C}$

(C)  $Y = (\bar{A} + \bar{B}) \bar{C}$

(D)  $Y = AB + C$

Answer:- (A)

Exp:-  $Y = \overline{A + B \cdot C} = \overline{A + B} + \bar{C} = \bar{A} \bar{B} + \bar{C}$

14. With initial condition  $x(1) = 0.5$ , the solution of the differential equation,

$t \frac{dx}{dt} + x = t$  is

(A)  $x = t - \frac{1}{2}$

(B)  $x = t^2 - \frac{1}{2}$

(C)  $x = \frac{t^2}{2}$

(D)  $x = \frac{t}{2}$

Answer:- (D)

Exp:- Given DE is  $t \frac{dx}{dt} + x = t \Rightarrow \frac{dx}{dt} + \frac{x}{t} = 1$

IF =  $e^{\int \frac{1}{t} dt} = e^{\log t} = t$ ; solution is  $x(IF) = \int (IF) t dt$

$xt = \int t \cdot t dt \Rightarrow xt = \frac{t^2}{2} + c$ ; Given that  $x(1) = 0.5 \Rightarrow 0.5 = \frac{1}{2} + c \Rightarrow c = 0$

$\therefore$  the required solution is  $xt = \frac{t^2}{2} \Rightarrow x = \frac{t}{2}$

15. The unilateral Laplace transform of  $f(t)$  is  $\frac{1}{s^2 + s + 1}$ . The unilateral Laplace transform of  $tf(t)$  is

(A)  $-\frac{s}{(s^2 + s + 1)^2}$

(B)  $-\frac{2s + 1}{(s^2 + s + 1)^2}$

(C)  $\frac{s}{(s^2 + s + 1)^2}$

(D)  $\frac{2s + 1}{(s^2 + s + 1)^2}$

Answer:- (D)

Exp:- If  $f(t) \leftrightarrow F(s)$ , then  $tf(t) \leftrightarrow -\frac{d}{ds}F(s)$

Thus if  $F(s) = \frac{1}{s^2 + s + 1}$

$tf(t) \rightarrow -\frac{d}{ds}\left(\frac{1}{s^2 + s + 1}\right) = \frac{2s + 1}{s^2 + s + 1}$

16. The average power delivered to an impedance  $(4 - j3)\Omega$  by a current  $5\cos(100\pi t + 100)$  A is

(A) 44.2W

(B) 50W

(C) 62.5W

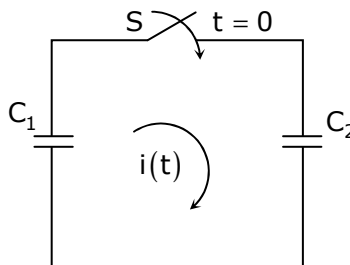
(D) 125W

Answer:- (B)

Exp:-  $Z = 4 - j3 = R_L - jX_C$ ;  $R_L = 4$ ;  $I = 5\cos(100\pi t + 100) = I_m \cos(\omega t + \alpha)$

$P = \frac{1}{2}I_m^2 R_L = \frac{1}{2} \times 5^2 \times 4 = 50W$

17. In the following figure,  $C_1$  and  $C_2$  are ideal capacitors.  $C_1$  has been charged to 12 V before the ideal switch S is closed at  $t = 0$ . The current  $i(t)$  for all  $t$  is



(A) zero

(B) a step function

(C) an exponentially decaying function

(D) an impulse function

Answer:- (D)

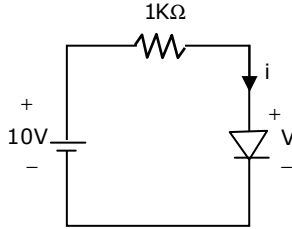
Exp:- When the switch is closed at  $t = 0$

Capacitor  $C_1$  will discharge and  $C_2$  will get charge since both  $C_1$  and  $C_2$  are ideal and there is no-resistance in the circuit charging and discharging time constant will be zero.

Thus current will exist like an impulse function.

18. The i-v characteristics of the diode in the circuit given below are

$$i = \begin{cases} \frac{v - 0.7}{500} \text{ A, } v \geq 0.7\text{V} \\ 0 \text{ A, } v < 0.7\text{V} \end{cases}$$



The current in the circuit is

- (A) 10mA                      (B) 9.3mA                      (C) 6.67mA                      (D) 6.2mA

Answer:- (D)

$$i = \frac{V - 0.7}{500}$$

$$\frac{di}{dV} = \frac{1}{500}$$

$$\Rightarrow r_d = 500\Omega$$

Since diode will be forward biased voltage across diode will be 0.7V

$$i = \frac{10 - 0.7}{1000 + 500} = \frac{9.3}{1500} = 6.2 \text{ mA}$$

19. The output Y of a 2-bit comparator is logic 1 whenever the 2-bit input A is greater than the 2-bit input B. The number of combinations for which the output is logic 1, is

- (A) 4                      (B) 6                      (C) 8                      (D) 10

Answer:- (B)

Exp:-

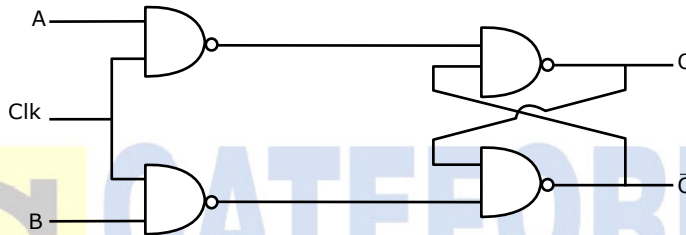
Input A		Input B		Y
A <sub>2</sub>	A <sub>1</sub>	B <sub>2</sub>	B <sub>1</sub>	
0	0	0	0	.....0
0	0	0	1	.....0
0	0	1	0	.....0
0	0	1	1	.....0
0	1	0	0	.....1



0	1	0	1.....0
0	1	1	0.....0
0	1	1	1.....0
1	0	0	0.....1
1	0	0	1.....1
1	0	1	0.....0
1	0	1	1.....0
1	1	0	0.....1
1	1	0	1.....1
1	1	1	0.....1
1	1	1	1.....0

Thus for 6 combinations output in logic 1

20. Consider the given circuit

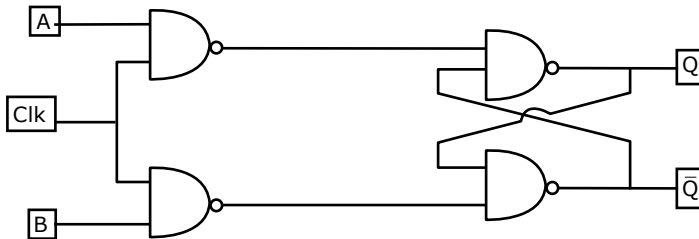


In this circuits, the race around

- (A) does not occur
- (B) occurs when CLK = 0
- (C) occur when CLK = 1 and A = B = 1
- (D) occurs when CLK = 1 and A = B = 0

Answer:- (A)

Exp:- .



$$Q_{next} = \overline{\overline{A \cdot CLK} \cdot \overline{Q}}$$

$$= A \cdot CLK + Q$$

$$\overline{Q}_{next} = A \cdot CLK + \overline{Q}$$

If CLK = 1 and A and B = 1

then  $\left. \begin{matrix} Q_{next} = 1 \\ \overline{Q}_{next} = 1 \end{matrix} \right\}$  No race around

If CLK = 1 and A = B = 0

$$\left. \begin{aligned} Q_{\text{next}} &= \overline{Q} \\ \overline{Q}_{\text{next}} &= Q \end{aligned} \right\} \text{No race around}$$

Thus race around does not occur in the circuit

21. The electric field of a uniform plane electromagnetic wave in free space, along the positive x direction, is given by  $\vec{E} = 10(\hat{a}_y + j\hat{a}_z)e^{-j25x}$ . The frequency and polarization of the wave, respectively, are
- (A) 1.2 GHz and left circular (B) 4 Hz and left circular  
(C) 1.2 GHz and right circular (D) 4 Hz and right circular

Answer:- (A)

Exp:-  $\frac{2\pi}{\lambda} = 25$

$$\Rightarrow \lambda = \left(\frac{2\pi}{25}\right)$$

$$f = \frac{3 \times 10^8}{\frac{2\pi}{25}} = 1.2 \text{ GHz}$$

Let  $E_y = \cos \omega t$

then  $E_z = \cos\left(\omega t + \frac{\pi}{2}\right)$

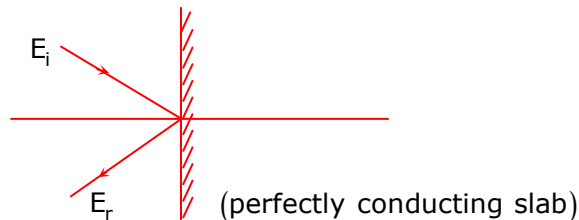
Now if we increase 't', we will see that it is in left circular polarization.



22. A plane wave propagating in air with  $\vec{E} = (8\hat{a}_x + 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t + 3x + 4y)}$  V/m is incident on a perfectly conducting slab positioned at  $x \leq 0$ . The  $\vec{E}$  field of the reflected wave is
- (A)  $(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t + 3x + 4y)}$  V/m  
(B)  $(-8\hat{a}_x + 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t + 3x + 4y)}$  V/m  
(C)  $(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t - 3x - 4y)}$  V/m  
(D)  $(-8\hat{a}_x + 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t - 3x - 4y)}$  V/m

Answer: (C)

$$E_i = (8\hat{a}_x + 6\hat{a}_y + 5\hat{a}_z)e^{j(\omega t + 3x + 4y)} \text{ V/m}$$



$x > 0 \quad x = 0 \quad x < 0$

1. Electric field inside a perfect conductor = 0  
∴ E<sub>transmitted</sub> = 0
  
2. For E<sub>i</sub> and E<sub>r</sub> y direction is same since the slab is  
Positioned at x = 0 and only x is reversed.  
∴ E<sub>i</sub> + E<sub>r</sub> = 0  
E<sub>r</sub> = E<sub>i</sub> = -(8ax + 6ay + 5az) e<sup>j(ωt - 3x - 4y)</sup>

23. In a baseband communications link, frequencies up to 3500 Hz are used for signaling. Using a raised cosine pulse with 75% excess bandwidth and for no inter-symbol interference, the maximum possible signaling rate in symbols per seconds is
- (A) 1750                      (B) 2625                      (C) 4000                      (D) 5250

Answer:- (C)

Exp:-  $B_T = \frac{1}{2} R_s (\beta + 1)$

R<sub>s</sub> → Symbol rate

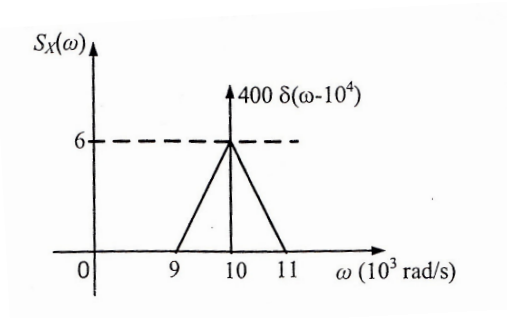
$$\Rightarrow R_s = \frac{2 \times \beta_T}{\beta + 1}$$

$$\beta = 0.75$$

$$\Rightarrow R_s = \frac{2 \times 3500}{1 + 0.75} = 4000 \text{ symbols / sec}$$

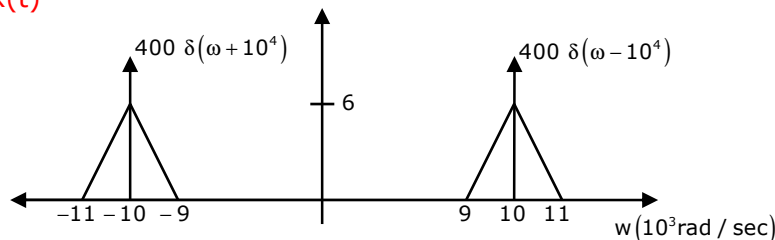
24. The power spectral density of a real process X(t) for positive frequencies is shown below. The values of E[X<sup>2</sup>(t)] and [E[X(t)]]<sup>2</sup>, respectively, are

- (A) 6000 / π, 0  
 (B) 6400 / π, 0  
 (C) 6400 / π, 20 / (π√2)  
 (D) 6000 / π, 20 / (π√2)



Answer:- (B)

Exp:- PSD of x(t)



$$E[x^2(t)] = R_{xx}(0)$$

$$R_{xx}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) d\omega$$

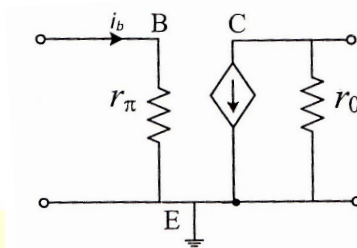
$$R_{xx}(\tau) \leftrightarrow S_{xx}(\omega)$$

fourier transform pair

$$= \frac{1}{2\pi} \left[ \frac{1}{2} \times 2 \times 10^3 \times 6 + \frac{1}{2} \times 2 \times 10^3 \times 6 + 400 + 400 \right] = 6400 / \pi$$

Since PSD of x(t) does not contain any DC component, the mean value of x(t) is zero.

25. The current  $i_b$  through the base of a silicon npn transistor is  $1 + 0.1 \cos(10000\pi t)$  mA. At 300 K, the  $r_\pi$  in the small signal model of the transistor is



(A) 250  $\Omega$

(B) 27.5  $\Omega$

(C) 25  $\Omega$

(D) 22.5  $\Omega$

Answer:- (C)

$$\text{Exp:- } r_\pi = \frac{V_T}{I_B} = \frac{25\text{mV}}{1\text{mA}} = 25\Omega$$

**Q. No. 26 – 55 Carry Two Marks Each**

26. Given that

$$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ the value of } A^3 \text{ is}$$

(A) 15A + 12I

(B) 19A + 30I

(C) 17A + 15I

(D) 17A + 21I

Answer:- (B)

$$\text{Exp :- Given: } A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix};$$

$$\text{Characteristic equation of A is } |A - I\lambda| = 0 \Rightarrow \begin{vmatrix} -5 - \lambda & -3 \\ 2 & 0 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (-5 - \lambda)(-\lambda) + 6 = 0 \Rightarrow 5\lambda + \lambda^2 + 6 = 0$$

$$\Rightarrow \lambda^2 = -5\lambda - 6 \text{ and } \lambda^3 = -5\lambda^2 - 6\lambda = -5(-5\lambda - 6) - 6\lambda$$

$$\lambda^3 = 25\lambda - 6\lambda + 30 = 19\lambda + 30$$

Every satisfies its characteristic equation

$$\therefore A^3 = 19A + 30I$$

27. The maximum value of  $f(x) = x^3 - 9x^2 + 24x + 5$  in the interval  $[1, 6]$  is  
 (A) 21 (B) 25 (C) 41 (D) 46

Answer:- (B)

EXP:- Given,  $f(x) = x^3 - 9x^2 + 24x + 5$

$f'(x) = 0$  for stationary values

$\Rightarrow 3x^2 - 18x + 24 = 0; \Rightarrow x = 2, 4; f''(x) = 6x - 18$

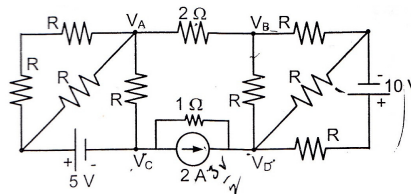
$f''(2) = 12 - 18 < 0; f''(4) = 24 - 18 > 0$

Hence  $f(x)$  has maximum value at  $x = 2$

$\therefore$  the maximum value is  $2^3 - 9 \times 2^2 + 24 \times 2 + 5 = 25$

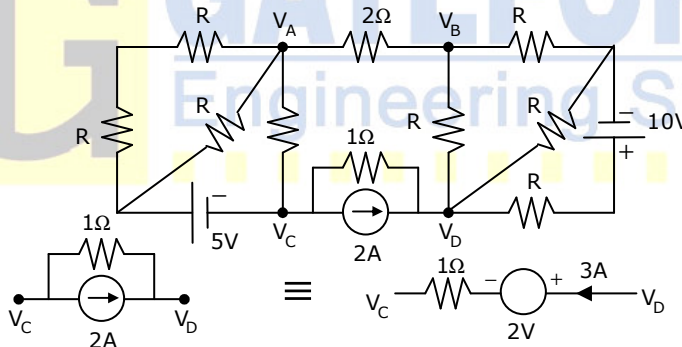
28. If  $V_A - V_B = 6$  V, then  $V_C - V_D$  is

- (A) -5V  
 (B) 2V  
 (C) 3V  
 (D) 6V



Answer:- (A)

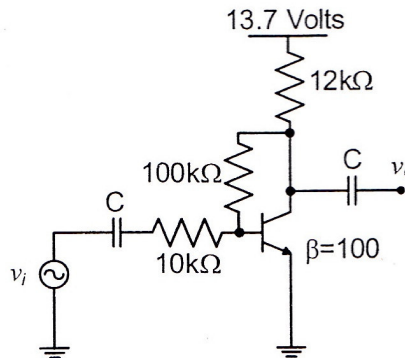
Exp:-  $I = \frac{V_A - V_B}{2} = \frac{6}{2} = 3A$ ; Since current entering any network is same as leaving in  $V_C - V_D$  branch also it is  $I = 3A$



$V_D = 2 + 3 + V_C = 5 + V_C; V_C - V_D = -5V$

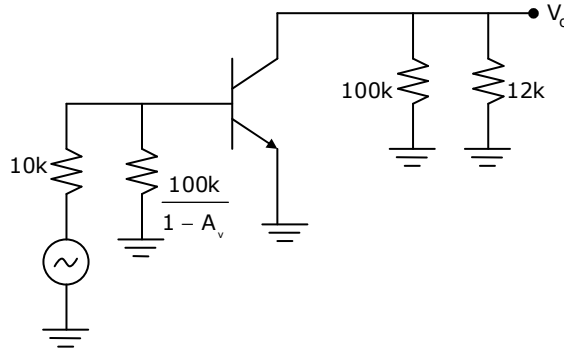
29. The voltage gain  $A_V$  of the circuit shown below is

- (A)  $|A_V| \approx 200$   
 (B)  $|A_V| \approx 100$   
 (C)  $|A_V| \approx 20$   
 (D)  $|A_V| \approx 10$



Answer:- (D)

Exp:-



KVL in input loop,  $13.7 - (I_C + I_B)12k - 100k(I_B) - 0.7 = 0$

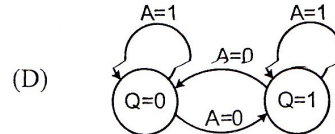
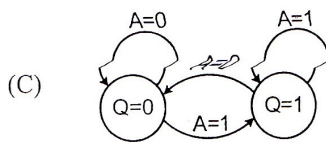
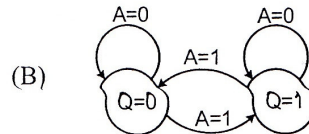
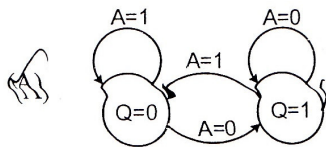
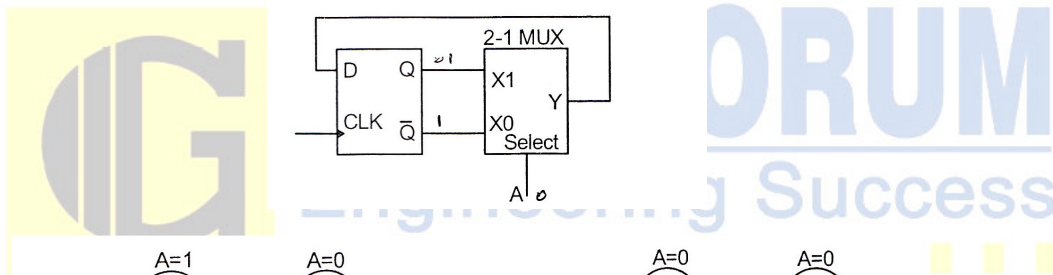
$\Rightarrow I_B = 9.9\mu A; I_C = \beta I_B = 0.99mA; I_E = 1mA$

$\therefore r_e = \frac{26mV}{I_E} = 26\Omega; z_i = \beta r_e = 2.6k\Omega; \therefore A_v = \frac{(100k \parallel 12k)}{26} = 412$

$z_i' = z_i \parallel \left( \frac{100k}{1 + 412} \right) = 221\Omega; A_{vs} = A_v \frac{z_i'}{z_i' + R_s} = (412) \left( \frac{221}{221 + 10k} \right)$

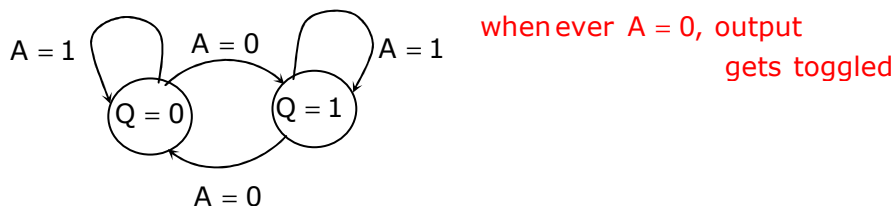
$|A_{vs}| \approx 10$

30. The state transition diagram for the logic circuit shown is



Answer:- (D)

Exp:-  $\left. \begin{matrix} A = 0, & y = Q \\ A = 1, & y = \bar{Q} \end{matrix} \right\}$  when ever  $A = 1$ , output gets into same state



31. Let  $y[n]$  denote the convolution of  $h[n]$  and  $g[n]$ , where  $h[n] = (1/2)^n u[n]$  and  $g[n]$  is a causal sequence. If  $y[0] = 1$  and  $y[1] = 1/2$ , then  $g[1]$  equals  
 (A) 0 (B) 1/2 (C) 1 (D) 3/2

Answer:- (A)

Exp:-

$$y[n] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k g[n-k]$$

$$y[0] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k g[-k] = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^0 g(0) = 1$$

$$\Rightarrow g(0) = 1$$

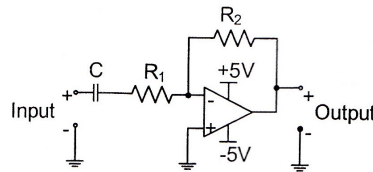
Since  $g(n)$  is Causal sequence  
 $g(-1), g(-2), \dots = 0$

$$y[1] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k g[1-k]$$

$$\Rightarrow \left(\frac{1}{2}\right)^0 g[1] + \left(\frac{1}{2}\right)^1 g(0) = \frac{1}{2}$$

$$g[1] = 0$$

32. The circuit shown is a



(A) low pass filter with  $f_{3dB} = \frac{1}{(R_1 + R_2)C}$  rad/s

(B) high pass filter with  $f_{3dB} = \frac{1}{R_1 C}$  rad/s

(C) low pass filter with  $f_{3dB} = \frac{1}{R_1 C}$  rad/s

(D) high pass filter with  $f_{3dB} = \frac{1}{(R_1 + R_2)C}$  rad/s

Answer:- (B)

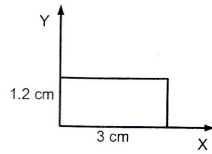
$$\text{Exp:- } V_0(S) = - \left( \frac{R_2}{R_1 + \frac{1}{CS}} \right) v_1(s)$$

$$V_0(S) = - \frac{R_2 CS}{(R_1 CS + 1)} V_i(S)$$

Thus cutoff frequency is  $\frac{1}{R_1 C}$  and the filter is high pass filter

33. The magnetic field along the propagation direction inside a rectangular waveguide with the cross section shown in the figure is

$$H_z = 3 \cos(2.094 \times 10^2 x) \cos(2.618 \times 10^2 y) \cos(6.283 \times 10^{10} t - \beta z)$$



The phase velocity  $V_p$  of the wave inside the waveguide satisfies

- (A)  $v_p > c$                       (B)  $v_p = c$                       (C)  $0 < v_p < c$                       (D)  $v_p = 0$

Answer:- (D)

Exp:-  $H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \cos(\omega t - \beta z)$

Given,

$$H_z = 3 \cos(2.094 \times 10^2 x) \cos(2.618 \times 10^2 y) \cos(6.283 \times 10^{10} t - \beta z)$$

Comparing two equations,

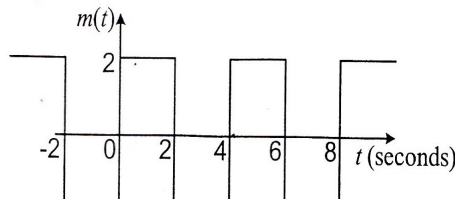
$$\text{we get } m = 2, n = 1$$

$$f = 10 \text{ GHz}$$

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3 \times 10^{10}}{2} \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{1.2}\right)^2} = 1.5 \text{ GHz}$$

Thus wave will not propagate inside waveguide hence  $V_p = 0$

34. The signal  $m(t)$  as shown is applied both to a phase modulator (with  $k_p$  as the phase constant) and a frequency modulator (with  $k_f$  as the frequency constant) having the same carrier frequency



The ratio  $k_p/k_f$  (in rad/Hz) for the same maximum phase deviation is

- (A)  $8\pi$                       (B)  $4\pi$                       (C)  $2\pi$                       (D)  $\pi$

Answer:- (B)

Exp:- In phase modulation,

$$\text{Maximum Phase deviation} = K_p |m(t)|_{\max} = K_p \cdot 2$$

In Frequency modulation,

$$\text{Maximum Phase deviation} = 2\pi k_f \int_a^2 dt = 2\pi k_f \times 4$$

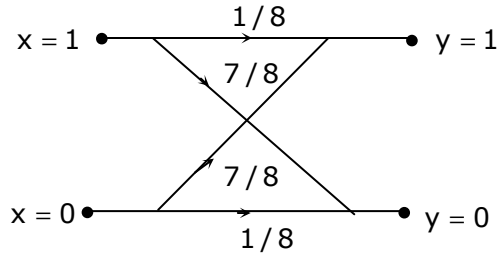


Now  $K_p \cdot 2 = 2\pi K_f \times 4 \Rightarrow \frac{K_p}{K_f} = 4\pi$

35. A binary symmetric channel (BSC) has a transition probability of  $1/8$ . If the binary transmit symbol  $X$  is such that  $P(X = 0) = 9/10$ , then the probability of error for an optimum receiver will be  
 (A)  $7/80$                       (B)  $63/80$                       (C)  $9/10$                       (D)  $1/10$

**Answer:- No option Matching**

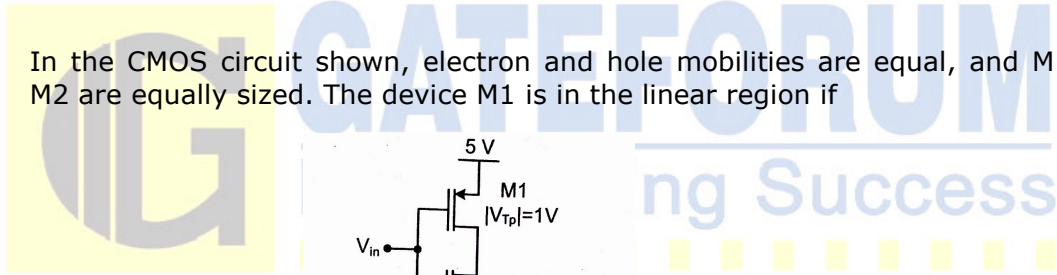
Exp:-



$$p(e) = p(0) p\left(\frac{1}{0}\right) + p(1) p\left(\frac{0}{1}\right)$$

$$= \frac{9}{10} \times \frac{7}{8} + \frac{1}{10} \times \frac{7}{8} = \frac{63}{80} + \frac{7}{80} = \frac{70}{80} = \frac{7}{8}$$

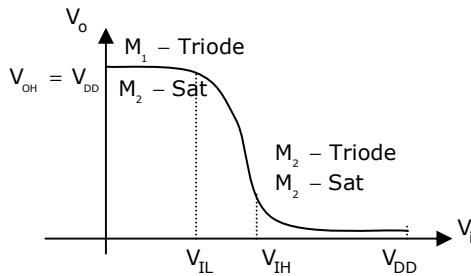
36. In the CMOS circuit shown, electron and hole mobilities are equal, and M1 and M2 are equally sized. The device M1 is in the linear region if



- (A)  $V_{in} < 1.875V$                       (B)  $1.87V < V_{in} < 3.125V$   
 (C)  $V_{in} > 3.125V$                       (D)  $0 < V_{in} < 5V$

**Answer:- (A)**

Exp:- The voltage transfer characteristics of the CMOS is:



where  $V_{IL} = \frac{1}{8} (3V_{DD} + 2V_t) = 2.125V$ , Hence Option 'A' is approximately correct.

37. A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is  
 (A)  $1/3$  (B)  $1/2$  (C)  $2/3$  (D)  $3/4$

Answer:- (C)

Exp:-  $P(\text{odd tosses}) = P(H) + P(TTH) + P(TTTTH) + \dots$

$$\begin{aligned} &= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \dots \\ &= \frac{1}{2} \left( 1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \dots \right) \\ &= \frac{1}{2} \left[ 1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \dots \right] = \frac{1}{2} \left( \frac{1}{1 - \frac{1}{4}} \right) = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3} \end{aligned}$$

38. The direction of vector A is radially outward from the origin, with  $|A| = kr^n$  where  $r^2 = x^2 + y^2 + z^2$  and k is a constant. The value of n for which  $\nabla \cdot A = 0$  is  
 (A) -2 (B) 2 (C) 1 (D) 0

Answer:- (D)

Exp:- Given :  $r^2 = x^2 + y^2 + z^2 \Rightarrow r = (x^2 + y^2 + z^2)^{1/2}$

$$|A| = kr^n = k(x^2 + y^2 + z^2)^{n/2} \Rightarrow A = (i + j + k)k(x^2 + y^2 + z^2)^{n/2}$$

$$\nabla \cdot A = 0 \Rightarrow \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot (i + j + k)k(x^2 + y^2 + z^2)^{n/2} = 0$$

$$\Rightarrow \sum k(x^2 + y^2 + z^2)^{n/2} = 0 \Rightarrow \frac{n}{2} \left[ (x^2 + y^2 + z^2)^{\frac{n}{2}-1} \right] [2x + 2y + 2z] = 0$$

$$\Rightarrow n = 0$$

39. Consider the differential equation

$$\frac{d^2y(t)}{dt^2} + 2 \frac{dy(t)}{dt} + y(t) = \delta(t) \text{ with } y(t)|_{t=0^-} = -2 \text{ and } \frac{dy}{dt}|_{t=0^-} = 0$$

The numerical value of  $\frac{dy}{dt}|_{t=0^+}$  is

- (A) -2 (B) -1 (C) 0 (D) 1

Answer:- (D)

Exp:-  $\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + y(t) = \delta(t)$

Converting to s – domain,

$$s^2y(s) - sy(0) - y'(0) + 2[sy(s) - y(0)] + y(s) = 1$$

$$[s^2 + 2s + 1]y(s) + 2s + 4 = 1$$

$$y(s) = \frac{-3 - 2s}{(s^2 + 2s + 1)}$$

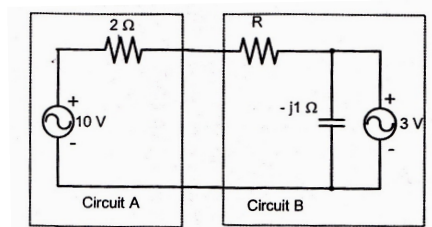
Find inverse lapalce transform

$$y(t) = [-2e^{-t} - te^{-t}]u(t)$$

$$\frac{dy(t)}{dt} = 2e^{-t} + te^{-t} - e^{-t}$$

$$\left. \frac{dy(t)}{dt} \right|_{t=0^+} = 2 - 1 = 1$$

40. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



(A) 0.8 Ω

(B) 1.4 Ω

(C) 2 Ω

(D) 2.8 Ω

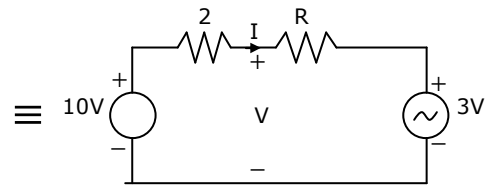
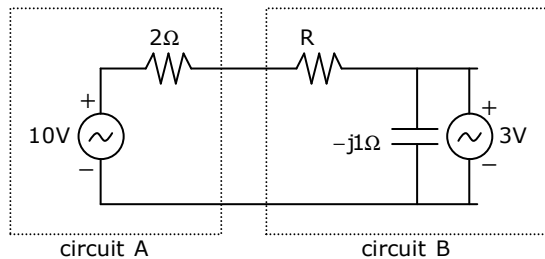
Answer:- (A)

Exp:- Power transferred from circuit A to circuit A =  $VI = \left(\frac{7}{R+2}\right)\left(\frac{6+10R}{R+2}\right) = \frac{42+70R}{(R+2)^2}$

$$I = \frac{10-3}{2+R} = \frac{7}{2+R}$$

$$V = 3 + IR = 3 + \frac{7R}{2+R} = \left(\frac{6+10R}{2+R}\right)$$

$$\frac{dP}{dR} = \frac{(R+2)^2(70) - (42+70R)2(R+2)}{(R+2)^4} = 0$$



$$\begin{aligned} 70(R+2)^2 &= (42+70R)2(R+2) \\ \Rightarrow 5(R+2) &= 2(3+5R) \\ \Rightarrow 5R+10 &= 6+10R \\ \Rightarrow 4 &= 5R \\ \Rightarrow R &= 0.8\Omega \end{aligned}$$

41. The state variable description of an LTI system is given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = (1 \quad 0 \quad 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where y is the output and u is the input. The system is controllable for

- (A)  $a_1 \neq 0, a_2 = 0, a_3 \neq 0$                       (B)  $a_1 = 0, a_2 \neq 0, a_3 \neq 0$   
 (C)  $a_1 = 0, a_2 \neq 0, a_3 = 0$                       (D)  $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

Answer:- (D)

Exp:- The controllability matrix

$$= [B \quad AB \quad A^2B]$$

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \text{controllability matrix} = \begin{bmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

for system to be controllable  
 determinant of controllability matrix should not be zero  
 $\Rightarrow a_1 \neq 0$   
 $a_2 \neq 0$   
 $a_3$  can be zero

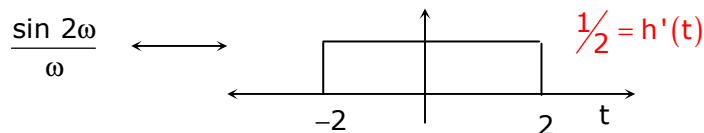


42. The fourier transform of a signal h(t) is  $H(j\omega) = (2 \cos \omega) (\sin 2\omega) / \omega$ . The value of h(0) is

- (A) 1/4                      (B) 1/2                      (C) 1                      (D) 2

Answer:- (C)

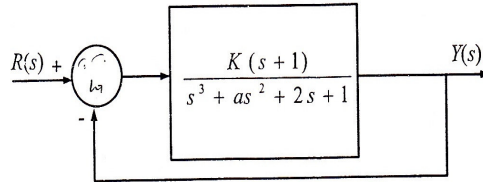
Exp:-



$$2 \cos \omega \left( \frac{\sin 2\omega}{\omega} \right) \longleftrightarrow h(t) = h'(t-1) + h'(t+1)$$

$$\left[ e^{j\omega} + e^{-j\omega} \right] \left( \frac{\sin 2\omega}{\omega} \right)$$

43. The feedback system shown below oscillates at 2 rad/s when



- (A)  $K = 2$  and  $a = 0.75$   
(C)  $K = 4$  and  $a = 0.5$

- (B)  $K = 3$  and  $a = 0.75$   
(D)  $K = 2$  and  $a = 0.5$

Answer:- (A)

Exp:-  $1 + G(S)H(S) = S^3 + as^2 + (2+k)s + 1+k$

$$\begin{array}{l} s^3 \quad 1(2+k) \\ s^2 \quad a(2+k) \\ s \quad a(2+k) - (2+k)0 \\ s^0 \quad (1+k)^a \end{array}$$

for system to oscillate

$$a(2+k) - (1+k) = 0$$

$$a = \left( \frac{1+k}{2+k} \right)$$

$$A.E \Rightarrow as^2 + (1+k) = 0 \Rightarrow s = \sqrt{\frac{1+k}{a}} = 2 \Rightarrow \left( \frac{1+k}{a} \right) = a \Rightarrow 2+k = 4 \Rightarrow k = 2$$

$$\text{Thus } a = 0.75$$

44. The input  $x(t)$  and output  $y(t)$  of a system are related as  $y(t) =$

$$\int_{-\infty}^t x(\tau) \cos(3\tau) d\tau. \text{ The system is}$$

- (A) time-invariant and stable  
(B) stable and not time-invariant  
(C) time-invariant and not stable  
(D) not time-invariant and not stable

Answer:- (B)

Exp:-  $y(t) = \int_{-\infty}^t x(\tau) \cos(3\tau) d\tau$

Since  $y(t)$  and  $x(t)$  are related with some function of time, so they are not time-invariant.

Let  $x(t)$  be bounded to some finite value  $k$ .

$$y(t) = \int_{-\infty}^t K \cos(3\tau) d\tau < \infty$$

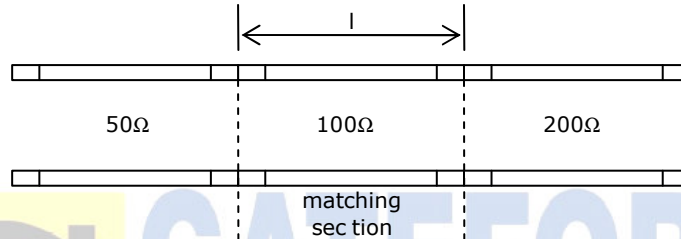
$y(t)$  is also bounded. Thus System is stable.

45. A transmission line with a characteristic impedance of  $100\Omega$  is used to match a  $50\Omega$  section to a  $200\Omega$  section. If the matching is to be done both at  $429\text{ MHz}$  and  $1\text{ GHz}$ , the length of the transmission line can be approximately  
 (A)  $82.5\text{ cm}$             (B)  $1.05\text{ m}$             (C)  $1.58\text{ m}$             (D)  $1.75\text{ m}$

Answer:- (B)

Exp:- Characteristic impedance =  $100\Omega$

sections impedance =  $50\Omega, 200\Omega$ ; frequency =  $429\text{MHz}, 1\text{GHz}$   
 (Matching section should have length  $l = \text{odd multiple of } \lambda/4$  where  $\lambda$  is operating wavelength)



for  $429\text{MHz}$ ,  $l_1 = \frac{\lambda_1}{4} = \frac{C}{4f_1} = 0.175\text{m}$ ;

for  $1\text{GHz}$ ,  $l_2 = \frac{\lambda_2}{4} = \frac{C}{4f_2} = 0.075\text{m}$

Length  $l$  should be integral multiples of both  $l_1$  and  $l_2$ .  
 $\therefore l = \text{multiple of LCM of } l_1 \text{ and } l_2 = \text{multiple of } 0.525\text{m}$   
 Hence,  $1.05\text{m}$  is the appropriate solution

46. A BPSK scheme operating over an AWGN channel with noise power spectral density of  $N_0/2$ , uses equiprobable signals  $s_1(t) = \sqrt{\frac{2E}{T}} \sin(\omega_c t)$  and  $s_2(t) = -\sqrt{\frac{2E}{T}} \sin(\omega_c t)$  over the symbol interval  $(0, T)$ . If the local oscillator in a coherent receiver is ahead in phase by  $45^\circ$  with respect to the received signal, the probability of error in the resulting system is

- (A)  $Q\left(\sqrt{\frac{2E}{N_0}}\right)$             (B)  $Q\left(\sqrt{\frac{E}{N_0}}\right)$             (C)  $Q\left(\sqrt{\frac{E}{2N_0}}\right)$             (D)  $Q\left(\sqrt{\frac{E}{4N_0}}\right)$

Answer:- (B)

Exp:- Given : BPSK scheme, AWGN channel with Noise power spectral density  $\frac{N_0}{2}$

Equiprobable signals

$$s_1(t) = \sqrt{\frac{2E}{T}} \sin(\omega_c t); \text{ and } s_2(t) = -\sqrt{\frac{2E}{T}} \sin(\omega_c t)$$

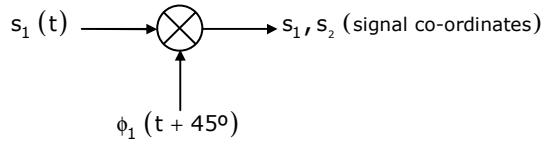
local oscillator in-coherent receiver is ahead in phase by  $45^\circ$  with respect to the received signal

Generally, we consider the local oscillator function of unit energy

$$\phi_1(t) = \sqrt{\frac{2}{T}} \sin \omega_c t \quad 0 \leq t \leq T \text{ but the local oscillator is ahead with } 45^\circ$$

$$\phi_1(t + 45^\circ) = \sqrt{\frac{2}{T}} \sin(\omega_c t + 45^\circ) \quad 0 \leq t \leq T$$

Receiver structure:



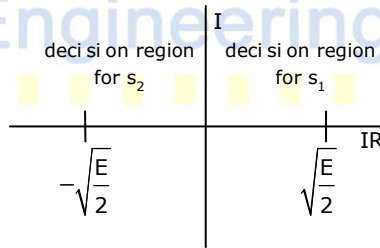
$$s_1 = \int_0^T s_1(t) \phi_1(t + 45^\circ) dt = \int_0^T \sqrt{\frac{2E}{T}} \sin \omega_c t \cdot \sqrt{\frac{2}{T}} \sin(\omega_c t + 45^\circ) dt$$

$$s_1 = \sqrt{\frac{E}{2}};$$

$$\text{similarly } s_2 = -\sqrt{\frac{E}{2}}$$

probability of making an error when we transmit  $\sqrt{\frac{E}{2}}$  i.e.  $s_1$

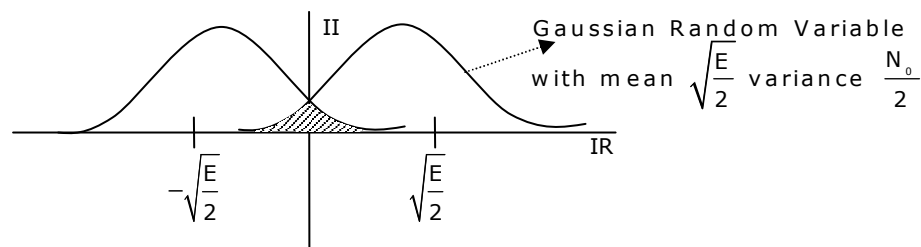
$$y = \sqrt{\frac{E}{2}} + N; \quad P\left(\frac{s_{2\text{received}}}{s_{1\text{transmitted}}}\right) = P(\gamma < 0) = P\left(\sqrt{\frac{E}{2}} + N < 0\right) = P\left(N < -\sqrt{\frac{E}{2}}\right)$$



probability of making an error when we transmit  $\sqrt{\frac{E}{2}}$  i.e.  $s_1$

$$y = \sqrt{\frac{E}{2}} + N;$$

$$P\left(\frac{s_{2\text{received}}}{s_{1\text{transmitted}}}\right) = P(\gamma < 0) = P\left(\sqrt{\frac{E}{2}} + N < 0\right) = P\left(N < -\sqrt{\frac{E}{2}}\right)$$



$$P\left(N < \sqrt{\frac{E}{2}}\right) = \int_{-\infty}^0 \frac{1}{\sqrt{2\pi\frac{N_0}{2}}} e^{-\frac{\left(x + \sqrt{\frac{E}{2}}\right)^2}{2\frac{N_0}{2}}} dx = \int_{-\infty}^0 \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(x + \sqrt{\frac{E}{2}}\right)^2}{N_0}} dx;$$

$$\text{Let } \frac{\left(x + \sqrt{\frac{E}{2}}\right)}{\sqrt{\frac{N_0}{2}}} = t; \quad dx = \sqrt{\frac{N_0}{2}} dt \Rightarrow P\left(N < \sqrt{\frac{E}{2}}\right) = \int_{\frac{\sqrt{E}}{\sqrt{N_0}}}^{\infty} \frac{1}{2\pi} e^{-\frac{t^2}{2}} dt = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

(∴ sign in the limit is removed since Area of Gaussian Pulse is same)

$$\text{Symbols are equiprobable } P(e) = \frac{1}{2} \left( P\left(\frac{S_{1\text{received}}}{S_{2\text{transmitted}}}\right) + P\left(\frac{S_{2\text{received}}}{S_{1\text{transmitted}}}\right) \right)$$

$$\therefore P(e) = \frac{1}{2} Q\left(\sqrt{\frac{E}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{E}{N_0}}\right) = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

47. The source of a silicon ( $n_i = 10^{10}$  per  $\text{cm}^3$ ) n-channel MOS transistor has an area of  $1 \text{ sq } \mu\text{m}$  and a depth of  $1 \mu\text{m}$ . If the dopant density in the source is  $10^{19} / \text{cm}^3$ , the number of holes in the source region with the above volume is approximately  
 (A)  $10^7$  (B) 100 (C) 10 (D) 0

Answer:- (D)

$$\text{Exp:- } p = \frac{n_i^2}{10^{19}} = 10 \text{ cm}^{-3}; \quad \text{volume} = 10^{-18} \text{ m}^3 = 10^{-12} \text{ cm}^3$$

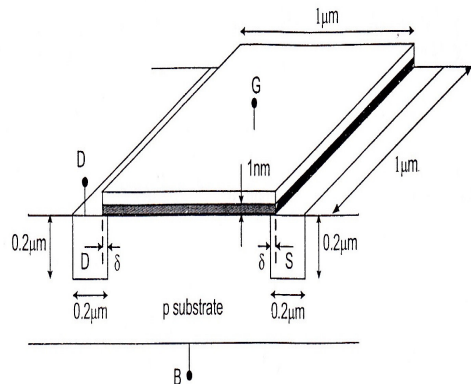
$$\therefore \text{total holes} = 10^{-11} \approx 0$$

**Common Data for Questions: 48 & 49**

In the three dimensional view of a silicon n-channel MOS transistor shown below,  $\delta = 20 \text{ nm}$

The transistor is of width  $1 \mu\text{m}$ .

The depletion width formed at every p-n junction is  $10 \text{ nm}$ . The relative permittivities of Si and  $\text{SiO}_2$ , respectively, are 11.7 and 3.9, and  $\epsilon_0 = 8.9 \times 10^{-12} \text{ F/m}$





48. The source-body junction capacitance is approximately  
 (A) 2fF (B) 7fF (C) 2pF (D) 7pF

Answer:- (A)

Exp:- Source – body junction capacitance =  $\frac{11.7 \times 1\mu \times 0.2\mu \times 8.9 \times 10^{-12}}{10\text{nm}} = 2\text{fF}$

49. The gate-source overlap capacitance is approximately  
 (A) 0.7fF (B) 0.7pF (C) 0.35fF (D) 0.24pF

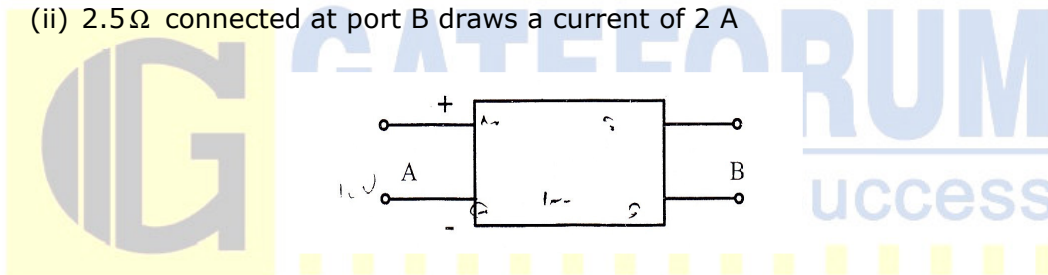
Answer:- (A)

Exp:-  $C = \frac{\epsilon A}{d}$ ;  $C_{\text{gsov}} = \frac{\epsilon(\text{SiO}_2) \times \text{Area}}{t_{\text{ox}}} = \frac{3.9 \times 8.9 \times 10^{-12} \times 1\mu \times 20}{1\text{nm}} = 0.7\text{pF}$

**Common Data for Questions: 50 & 51**

With 10V dc connected at port A in the linear nonreciprocal two-port network shown below, the following were observed:

- (i) 1Ω connected at port B draws a current of 3 A
- (ii) 2.5Ω connected at port B draws a current of 2 A



50. For the same network, with 6V dc connected at port A, 1Ω connected at port B draws 7/3 A. If 8V dc is connected to port A, the open circuit voltage at port B is  
 (A) 6V (B) 7V (C) 8V (D) 9V

Answer:- (B)

51. With 10V dc connected at port A, the current drawn by 7Ω connected at port B is  
 (A) 3/7A (B) 5/7A (C) 1 A (D) 9/7A

Answer:- (C)

Exp:- The given network can be replaced by a Thevenin equivalent with  $V_{\text{th}}$  and  $R_{\text{th}}$  as Thevenin voltage and Thevenin Resistance.

Now we can write two equations for this

$$V_{\text{th}} = 3R_{\text{th}} + 3$$

$$V_{\text{th}} = 2R_{\text{th}} + 5$$

Solving these two equations we get  $R_{\text{th}} = 2$  and  $V_{\text{th}} = 9$ .

Now using the same equation with current unknown,

$$9 = I \times 2 + 7 \times I \Rightarrow I = 1\text{A}$$

**Linked Answer Questions: Q.52 to Q.55 Carry Two Marks Each**

**Statement for Linked Answer Questions: 52 & 53**

The transfer function of a compensator is given as  $G_c(s) = \frac{s+a}{s+b}$

52.  $G_c(s)$  is a lead compensator if  
 (A)  $a = 1, b = 2$  (B)  $a = 3, b = 2$   
 (C)  $a = -3, b = -1$  (D)  $a = 3, b = 1$

Answer:- (A)

Exp:-  $\phi = \tan^{-1} \frac{\omega}{a} - \tan^{-1} \frac{\omega}{\beta}$

for phase lead  $\phi$  should be +ve

$\Rightarrow \tan^{-1} \frac{\omega}{a} > \tan^{-1} \frac{\omega}{\beta}$

$\Rightarrow a < b$

both option (A) and (C) satisfier

but option (C) will put polar and zero as

RHS of s-plane thus not possible

Option (A) is right

53. The phase of the above lead compensator is maximum at  
 (A)  $\sqrt{2}$  rad/s (B)  $\sqrt{3}$  rad/s  
 (C)  $\sqrt{6}$  rad/s (D)  $1/\sqrt{3}$  rad/s

Answer:- (A)

Exp:-  $\omega =$  geometric mass of two carrier frequencies

$= \sqrt{2 \times 1} = \sqrt{2}$  rad/sec

**Statement for Linked Answer Questions: 54 & 55**

An infinitely long uniform solid wire of radius  $a$  carries a uniform dc current of density  $\vec{j}$ .

54. The magnetic field at a distance  $r$  from the center of the wire is proportional to  
 (A)  $r$  for  $r < a$  and  $1/r^2$  for  $r > a$  (B)  $0$  for  $r < a$  and  $1/r$  for  $r > a$   
 (C)  $r$  for  $r < a$  and  $1/r$  for  $r > a$  (D)  $0$  for  $r < a$  and  $1/r^2$  for  $r > a$

Answer:- (C)

Exp:- For region  $0 \leq r \leq a$

using Ampere's circuital law

$$\int H \cdot dl = I_{enc} = \int J \cdot ds$$

$$I_{enc} = (J \cdot \pi r^2)$$

$$H \cdot (2\pi r) = J\pi r^2$$

$$H = \frac{J \cdot r}{2}$$

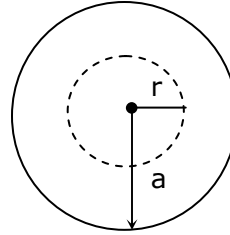
thus magnetic increases with r

for region  $r > a$

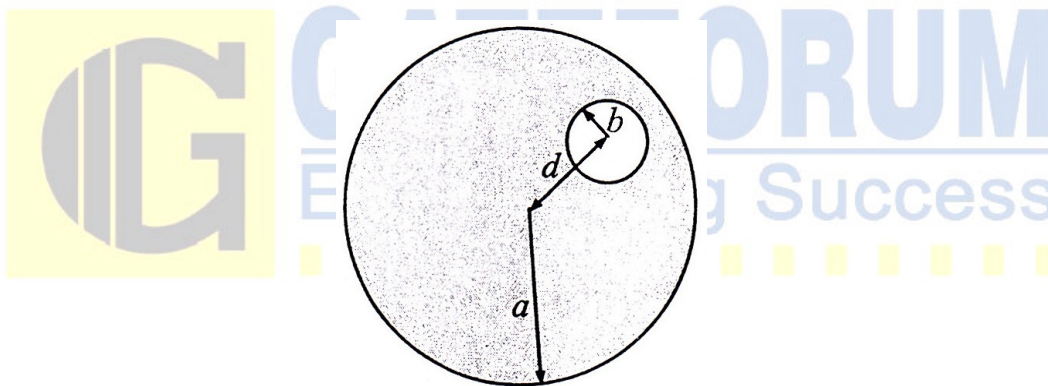
$$H_{\phi} (2\pi r) = J \cdot (\pi a^2)$$

$$H_{\phi} = \frac{J \cdot \pi a^2}{2\pi r} = \frac{J a^2}{2r}$$

Magnetic field is proportional to  $\frac{1}{r}$



55. A hole radius  $b$  ( $b < a$ ) is now drilled along the length of the wire at a distance  $d$  from the centre of the wire as shown below.



The magnetic field inside the hole is

- (A) uniform and depends only on  $d$
- (B) uniform and depends only on  $b$
- (C) uniform and depends on both  $b$  and  $d$
- (D) non uniform

Answer:- (B)

**Q. No. 56 –60 Carry One Mark Each**

56. Which one of the following options is the closest in meaning to the word given below?

**Latitude**

- (A) Eligibility
- (B) Freedom
- (C) Coercion
- (D) Meticulousness

Answer:- (B)

57. One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one the following is **INCORRECT**?

**I requested that he should be given the driving test today instead of tomorrow.**

- (A) requested that (B) should be given  
(C) the driving test (D) instead of tomorrow

Answer:- (B)

58. If  $(1.001)^{1259} = 3.52$  and  $(1.001)^{2062} = 7.85$ , then  $(1.001)^{3321} =$

- (A) 2.23 (B) 4.23 (C) 11.37 (D) 27.64

Answer:- (D)

Exp:- let  $1.001 = x$

$$x^{1259} = 3.52 \text{ and } x^{2062} = 7.85$$

$$x^{3321} = x^{1259} \cdot x^{2062} = 3.52 \times 7.85 = 27.64$$

59. Choose the most appropriate alternative from the options given below to complete the following sentence:

**If the tried soldier wanted to lie down, he \_\_\_\_\_ the mattress out on the balcony**

- (A) should take  
(B) shall take  
(C) should have taken  
(D) will have taken

Answer:- (C)

60. Choose the most appropriate word from the options given below to complete the following sentence:

**Given the seriousness of the situation that he had to face, his \_\_\_ was impressive.**

- (A) beggary  
(B) nomenclature  
(C) jealousy  
(D) nonchalance

Answer:- (D)

**Q. No. 61 –65 Carry Two Marks Each**

61. The data given in the following table summarizes the monthly budget of an average household.

Category	Amount (Rs)
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget **NOT** spent on saving is  
 (A) 10%                      (B) 14%                      (C) 81%                      (D) 86%

Answer:- (D)

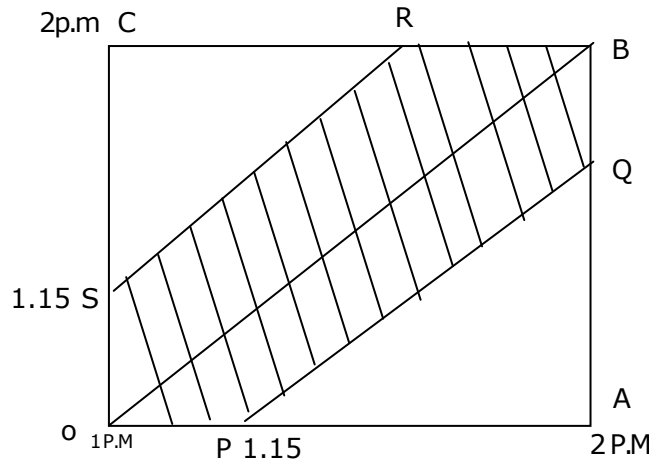
Exp:- Total budget = 10,500

Expenditure other than savings = 9000

Hence,  $\frac{9000}{10500} = 86\%$

62. A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is  
 (A) 1/4                      (B) 1/16                      (C) 7/16                      (D) 9/16

Answer:- (C)



OB is the line when both A and B arrive at same time.

Total sample space =  $60 \times 60 = 3600$

Favourable cases = Area of OABC – Area of PQRS

$$= 3600 - 2 \times \left( \frac{1}{2} \times 45 \times 45 \right) = 1575$$

$$\therefore \text{The required probability} = \frac{1575}{3600} = \frac{7}{16}$$

63. **One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.**

Which one of the following statements best sums up the meaning of the above passage?

- (A) Through regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.  
 (B) The legions were treated inhumanly as if the men were animals.  
 (C) Discipline was the armies' inheritance from their seniors.  
 (D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

Answer:- (A)

64. Raju has 14 currency notes in his pocket consisting of only Rs.20 notes and Rs. 10 notes. The total money value of the notes is Rs.230. The number of Rs. 10 notes that Raju has is

- (A) 5 (B) 6 (C) 9 (D) 10

Answer:- (A)

Exp:- Let the number of Rs. 20 notes be x and Rs. 10 notes be y

$$20x + 10y = 230$$

$$x + y = 14$$

$$x=9 \text{ and } y=5$$

Hence the numbers of 10 rupee notes are 5

65. There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighting balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is

- (A) 2 (B) 3 (C) 4 (4) 8

Answer:- (A)

Let us categorize the bags in three groups as

$$A_1 A_2 A_3 \quad B_1 B_2 B_3 \quad C_1 C_2$$

1<sup>st</sup> weighing A vs B

**Case -1**

$$A_1 A_2 A_3 = B_1 B_2 B_3$$

Then either  $C_1$  or  $C_2$  is heavier

**Case -2**

$$A_1 A_2 A_3 \neq B_1 B_2 B_3$$

Either A or B would be heavier (Say  $A > B$ )

**2<sup>nd</sup> weighing**

$C_1$  vs  $C_2$

If  $C_1 > C_2$ , then  $C_1$

If  $C_1 < C_2$ , then  $C_2$

If  $A_1 < A_2$ , then  $A_2$

$A_1$  vs  $A_2$

If  $A_1 = A_2$ , then  $A_3$

If  $A_1 > A_2$ , then  $A_1$

