Activities (Pages 231, 232)

1. Find the equation of the line joining (1, 2) and (2, 4). For points on this line with consecutive natural numbers 3, 4, 5, ... as x coordinates, what is the sequance of y coordinates?

On the line joining (1, 2) and (2, 4), as the x coordinate changes $2 - 1 = 1$, the y coordinate changes $4 - 2 = 2$. That means the change in y coordinate is 2 times the change in x coordinate.

Taking any point (x, y) on this line, the change in y from (1, 2) = 2 × change in x.

$\therefore y - 2 = 2(x - 1)$

$y - 2 = 2x - 2$

or $2x - y - 2 + 2 = 0$

or $2x - y = 0$. This is the equation of the line.

In (1, 2) and (2, 4), change in the x coordinates

$= 2 - 1 = 1$

In (1, 2) and (2, 4) change in the y coordinates

$= 4 - 2 = 2$

When x changes 1, y changes 2.

That means on this line, when x coordinates of the points are consecutive natural numbers like 1, 2, 3, 4, ... the y coordinates are consecutive even numbers like 2, 4, 6, 8, ...

2. **Find the equation of the line joining (–1, 3) and (2, 5). Prove that if the point (x, y) is on this line, so is the point (x + 3, y + 2).**

On the line joining (–1, 3) and (2, 5) as the x coordinate changes $2 - (-1) = 3$, the y coordinate changes $5 - 3 = 2$.

So the change in y is $\frac{2}{3}$ part of the change in x.

Taking any point (x, y) on this line, the change in y from (–1, 3) $= \frac{2}{3} \times$ change in x

$\therefore y - 3 = \frac{2}{3}(x - -1)$

$3(y - 3) = 2(x + 1)$

$3y - 9 = 2x + 2$

$2x - 3y + 2 + 9 = 0$

$2x - 3y + 11 = 0$. This is the equation of the line.

Taking any point (x, y) on this line as x changes 3, change in y is 2. Then corresponding to x + 3, it is y + 2. So the point (x + 3, y + 2) is also a point on that line.

3. **Prove that whatever number we take as x, the point (x, 2x + 3) is a point on the line joining (–1, 1) and (2, 7).**

On the line joining (–1, 1) and (2, 7) as the x coordinate changes $2 - (-1) = 3$, the y coordinate changes $7 - 1 = 6$.

So the change in y is $\frac{6}{3} = 2$ times the change in x.

Taking any point (x, y), on this line, the change in y from (–1, 1) $= 2 \times$ change in x.

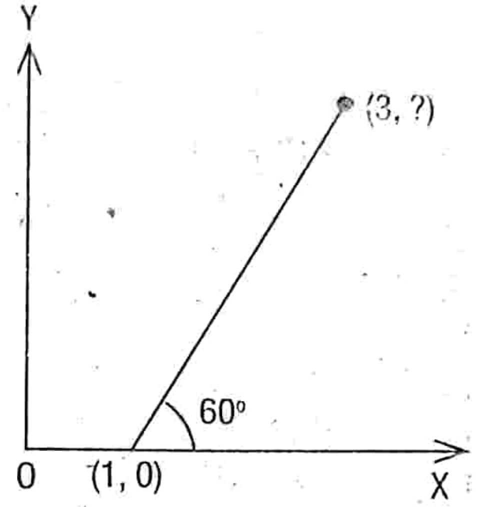
$y - 1 = 2 \times (x - -1) = 2(x + 1) = 2x + 2$

$y = 2x + 3$

That means taking any number as x coordinate on this line, the y coordinate is 2x + 3.

So (x, 2x + 3) is also a point on that line.

4. In the picture below, the x coordinate of a point on the slanted line is 3.

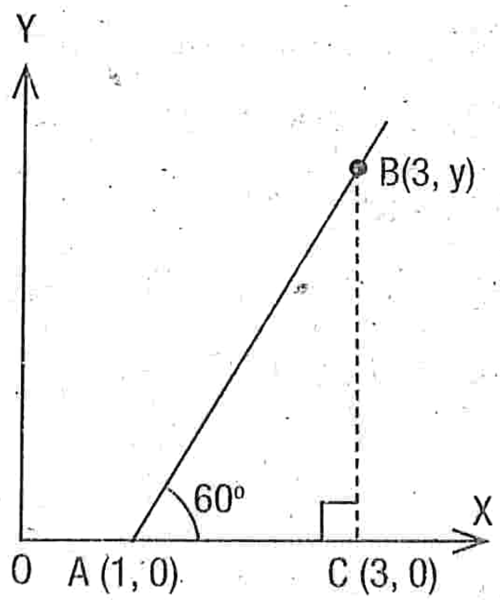


i) **What is its y coordinate?**

ii) **What is the slope of the line?**

iii) **Write the equation of the line.**

i) Let A(1, 0) and B(3, y). The perpendicular from B crosses the x axis at C.



x coordinate of C = x coordinate of B = 3

y coordinate of C = y coordinate of A = 0

Coordinates of C = (3, 0)

$AC = |3-1| = |2| = 2$

In ΔABC, $\angle B = 180 - (60 + 90) = 180 - 150 = 30^\circ$

Since the angles of ΔABC are 30°, 60° and 90°, sides opposite these angles are in the ratio $1 : \sqrt{3} : 2$

Since $AC = 2$, $BC = 2\sqrt{3}$.

Since $BC = 2\sqrt{3}$, y coordinate of $B = 2\sqrt{3}$

(ii) Slope of the line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2\sqrt{3} - 0}{3 - 1} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

(iii) Let (x, y) be a point on this line. Then the slope of the line containing (1, 0) and (x, y) and the slope of the line containing (1, 0) and $(3, 2\sqrt{3})$ will be the same.

Slope of the line containing (1, 0) and (x, y)

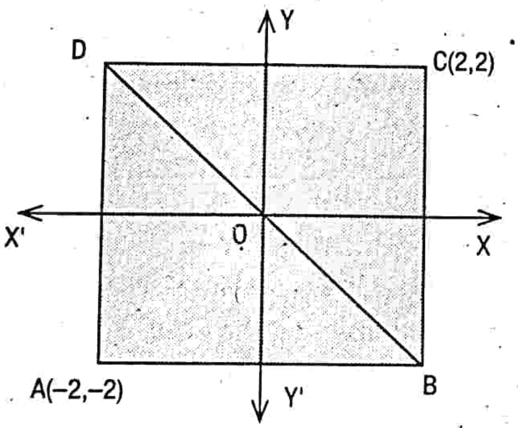
$$= \frac{y - 0}{x - 1} = \frac{y}{x - 1}$$

Since this slope is equal to $\sqrt{3}$, $\frac{y}{x-1} = \frac{\sqrt{3}}{1}$

$y = \sqrt{3}(x - 1)$

$y = \sqrt{3}x - \sqrt{3}$. This is the equation of the line.

5. In the picture here, ABCD is a square. Prove that for any point on the diagonal BD, the sum of the x and y coordinates is zero.



x coordinate of B = x coordinate of C = 2

y coordinate of B = y coordinate of A = –2

Coordinates of B = (2, –2)

In the same way coordinate of D = (–2, 2)

Slope of the line containing B and D

$$= \frac{2 - (-2)}{-2 - 2} = \frac{2 + 2}{-4} = \frac{4}{-4} = -1$$

Let (x, y) be a point on the line BD.

Then slope of the line containing (x, y) and (2, –2) is also –1.

That is $\frac{-2 - y}{2 - x} = \frac{-1}{1}$

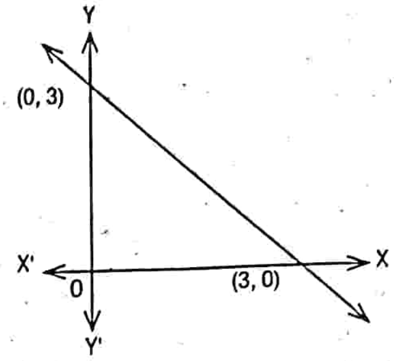
$-1(2 - x) = -2 - y$

$-2 + x = -2 - y$

$-2 + x + 2 + y = 0,\ x + y = 0$

That means the sum of the x coordinate and y coordinate is zero.

6. Prove that for any point on the line intersecting the axes in the picture, the sum of the x and y coordinates is 3.



Slope of the line containing (0, 3) and (3, 0)

$$= \frac{0 - 3}{3 - 0} = \frac{-3}{3} = -1$$

Let (x, y) be a point on this line. Then slope of the line containing (0, 3) and (x, y) is also –1.

$\frac{y - 3}{x - 0} = \frac{-1}{1}$

$-1(x) = 1(y - 3)$

$-x = y - 3,\ -x - y + 3 = 0$

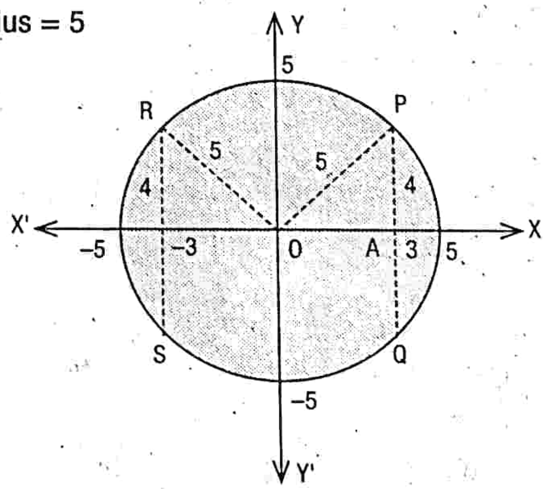
$x + y - 3 = 0,\ x + y = 3$

That means the sum of the x and y coordinate is 3.

7. Find the equation of the circle with centre at the origin and radius 5. Write the coordinates of eight points on this circle.

Centre of the circle = (0,0)

Radius = 5



Taking any point (x, y) on the circle, the distance between (x, y) and the centre of the circle is the radius of the circle.

Distance between (x, y) and the origin (0, 0)

$$= \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

$\therefore \sqrt{x^2 + y^2} = 5$ (radius = 5)

$\therefore x^2 + y^2 = 5^2$

or $x^2 + y^2 = 25$. This is the equation of the circle.

Points where the circle crosses the x axis = (5, 0), (–5, 0)

Points where the circle crosses the y axis = (0, 5), (0, –5)

If OA = 3, OP = 5

Since 3, 4, 5 is a Pythagorean triplet, AP = 4

Coordinates of P = (3, 4)

Coordinates of Q = (3, –4)

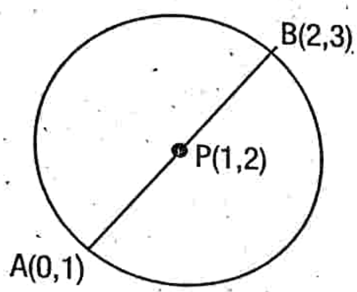
Coordinates of R = (–3, 4)

Coordinates of S = (–3, –4)

8. Prove that if (x, y) be a point on the circle with the line joining (0, 1) and (2, 3) as diameter, then $x^2 + y^2 - 2x - 4y + 3 = 0$. Find the coordinates of the points where this circle cuts the y axis.

Let A(0, 1) and B(2, 3) be the endpoints of the diameter of the circle.

The centre of the circle is the midpoint P of AB.



Coordinates of P

$$= \left(\frac{0+2}{2}, \frac{1+3}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{4}{2}\right) = (1, 2)$$

Radius of the circle, $AP = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(1-0)^2 + (2-1)^2}$$

$$= \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

Let (x, y) be any point on the circle. The square of the distance between (x, y) and the centre of the circle (1, 2) = square of the radius.

$\therefore (x-1)^2 + (y-2)^2 = (\sqrt{2})^2$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 2$

$x^2 + y^2 - 2x - 4y + 1 + 4 - 2 = 0$

$\therefore x^2 + y^2 - 2x - 4y + 3 = 0$

This is the equation of the circle.

Let this circle cut the y axis at (0, b). (x coordinate on the y axis is 0)

In the equation of the circle, put x = 0 and y = b.

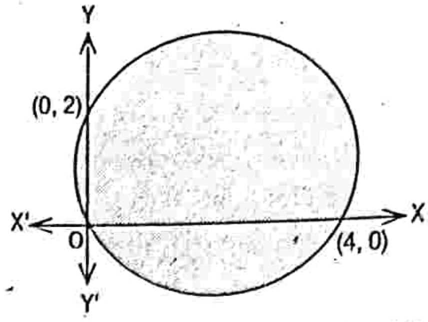
$0 + b^2 - 0 - 4b + 3 = 0$

ie $b^2 - 4b + 3 = 0$

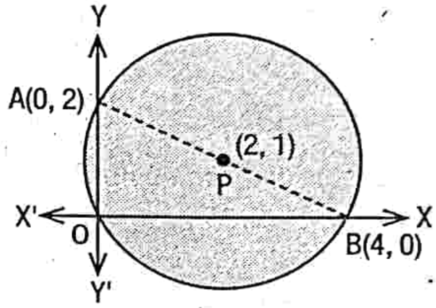
ie $(b-1)(b-3) = 0$. $\therefore b = 1$ or 3

$\therefore$ Coordinates of the points where this circle cuts the y axis = (0, 1) and (0, 3)

9. What is the equation of the circle shown here?



Given that the circle cuts the y axis at the point A(0,2) and cuts the x axis at the point B(4,0). Since the axis are perpendicular to each other, $\angle YOX = 90^\circ$.



We have learnt that angle in a semicircle is a right angle. So AB is the diameter of the circle. Its midpoint P is the centre of the circle.

Coordinates of $P = \left(\frac{0+4}{2}, \frac{2+0}{2}\right) = (2, 1)$

Radius of the circle,

$$AP = \sqrt{(2-0)^2 + (1-2)^2}$$

$$= \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$$

Let (x, y) be any point on the circle.

The square of the distance between (x, y) and the centre (2, 1) of the circle = square of the radius.

$\therefore (x-2)^2 + (y-1)^2 = (\sqrt{5})^2$

$x^2 - 4x + 4 + y^2 - 2y + 1 = 5$

$x^2 + y^2 - 4x - 2y + 4 + 1 - 5 = 0$

or $x^2 + y^2 - 4x - 2y = 0$

This is the equation of the circle.