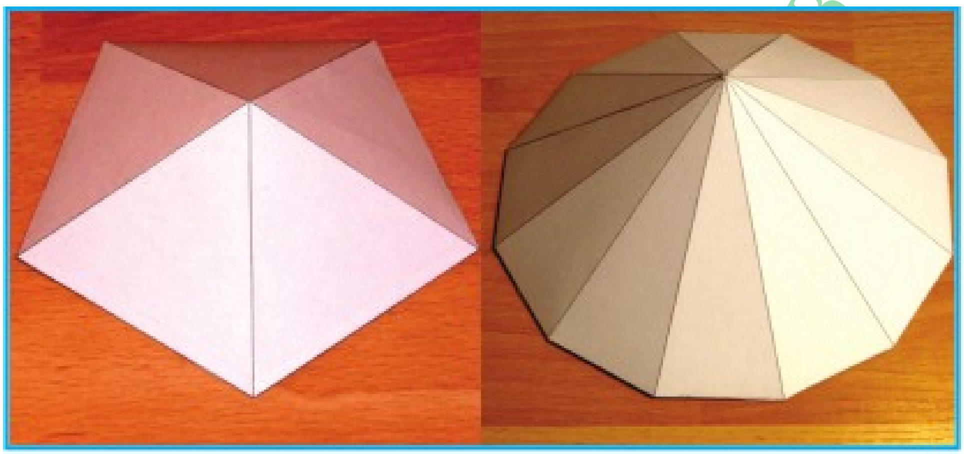
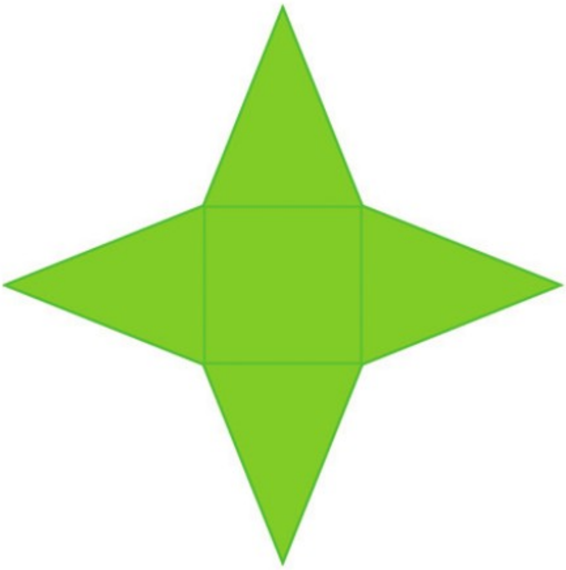
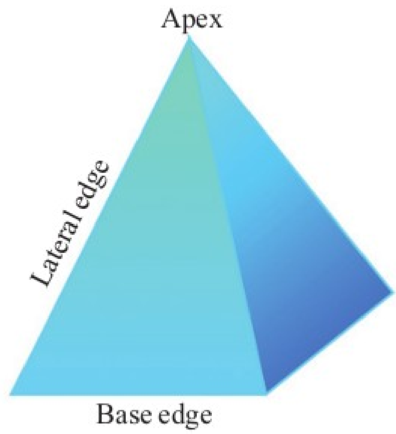
Pyramids



Cut out a figure like the following in a thick paper. A square in the middle and four triangles around it; all four of them are isosceles triangles and they are equal. Now fold and paste as shown.

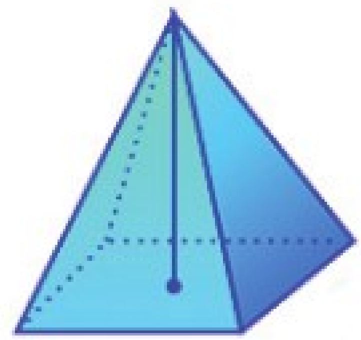


Instead of square, the base can be some other rectangle, a triangle or some other polygon. Such solids are called pyramids.



The sides of the polygon forming the base of a pyramid are called base edges and the other sides of the triangles are called lateral edges. The topmost point of a pyramid is called its apex. Usually base edge is represented by ‘a’ and lateral edge by ‘e’

Height of a pyramid



The height of a pyramid is the perpendicular distance from the apex to the base. Usually height is represented by ‘h’

Square pyramid

A square pyramid has 5 faces. Base is a square and other four faces are isosceles acute angled triangles. These are called lateral faces. All lateral faces are equal.

Note: When the base edges and lateral edges are equal, lateral faces become equilateral triangles.

A square pyramid has 8 edges. 4 base edges and 4 lateral edges. It has 5 vertices in which one is the apex.

Activity

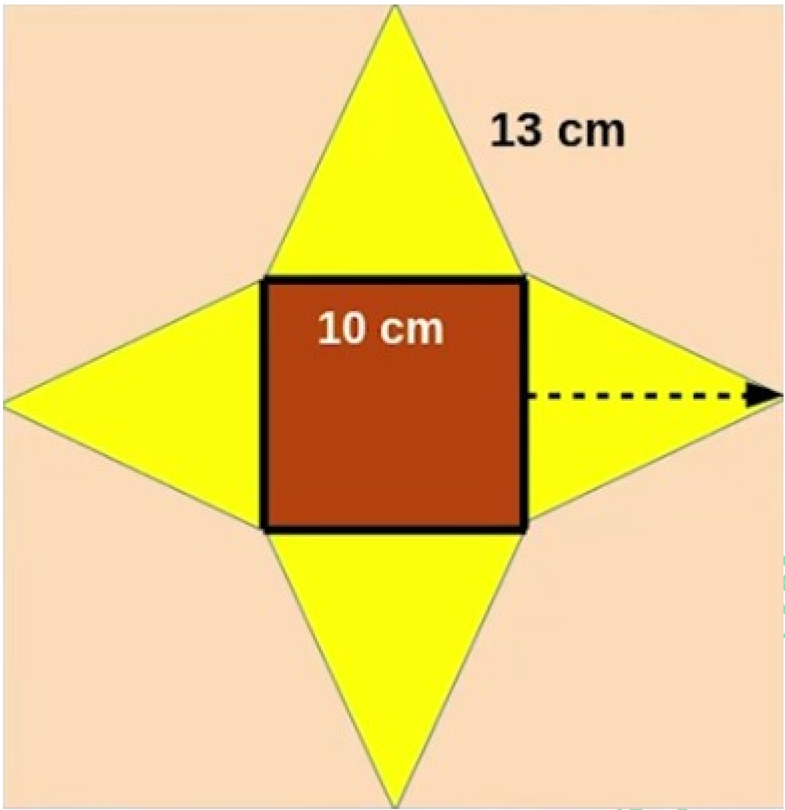
Jishnu made a square pyramid from a square paper sheet. Its base edge is 10 cm and lateral edge is 13 cm. Then

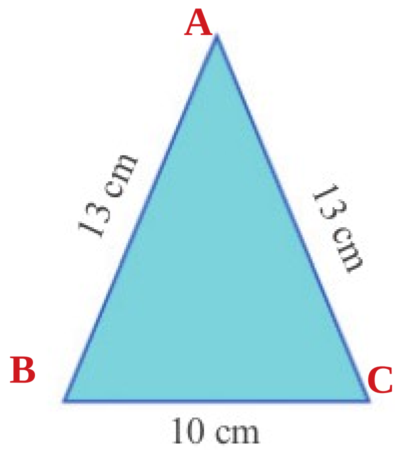
a) what is the side of the square paper sheet Jishnu used to make the square pyramid?

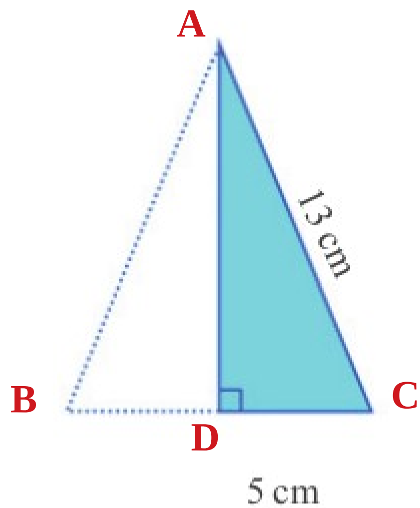
b) What is the surface area of the square pyramid?

Answer

a)







Base edge, BC = 10 cm

Lateral edge, AB = AC = 13 cm

We have to find the height of one lateral face(AD).

ΔADC is a right triangle.

CD = 5 cm

AD = $\sqrt{AC^2 - CD^2} = \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25} = \sqrt{144}$ **= 12 cm**

Therefore,

Side of the large square paper sheet = 12 + 10 + 12 = 34 cm

Note

The height of triangle is the slant height or lateral height of the square pyramid. It is usually denoted by 'l'

That is, $(\text{slant height})^2 = (\text{lateral edge})^2 - \left(\frac{base\ edge}{2}\right)^2$

$$l^2 = e^2 - \left(\frac{a}{2}\right)^2$$

b) Base area = $a^2 = 10^2 = 100\ cm^2$

Area of one lateral face $= \frac{1}{2} \times base\ edge \times lateral\ height$

$= \frac{1}{2} \times 10 \times 12$

$= 60\ cm^2$

Area of such 4 lateral faces $= 4 \times 60 = 240\ cm^2$

Therefore,

Total surface area of square pyramid $= 100 + 240 = 340\ cm^2$

Note:

In a square pyramid,

Base area, A = $(base\ edge)^2 = a^2$

Area of one lateral face = $\frac{1}{2} \times base\ edge \times lateral\ height = \frac{1}{2} \times a \times l$

Lateral surface area = $4 \times \frac{1}{2} \times a \times l = 2 \times a \times l = 2al$

Total surface area = $a^2 + 2al$

Assignment:

A square of side 5 centimetres, and four isosceles triangles of base 5 centimetres and height 8 centimetres, are to be put together to make a square pyramid. How many square centimetres of paper is needed?

Prepared by Jaisingh Jose G R ;HST(Maths) Govt.V&HSS Kulathoor

Assignment on 15-12-2020

A square of side 5 centimetres, and four isosceles triangles of base 5 centimetres and height 8 centimetres, are to be put together to make a square pyramid. How many square centimetres of paper is needed?

Answer

Base edge, a = 5 cm
Slant height, l = 8 cm
Total area of paper needs to make the square pyramid = Total surface area of the square pyramid = $a^2 + 2al$

$$= 5^2 + 2 \times 5 \times 8 = 25 + 80 = 105 \text{ cm}^2$$

Note:

In square pyramid, the lateral faces are always equal acute angled isosceles triangles. Otherwise, The height of one isosceles triangle should always be greater than half the base edge.

That is, slant height(l) > half of base edge($\frac{a}{2}$)

That is, $l > \frac{a}{2}$

Base area and lateral surface area of a square pyramid cannot be equal. Base area is always less than the lateral surface area.

That is, $a^2 < 2al$

In a square pyramid, if all edges are equal; then lateral faces are equilateral triangles.

Therefore, Area of one lateral face = Area of an equilateral triangle with side 'a' = $\frac{\sqrt{3}}{4} \times a^2$

Lateral surface area = $\frac{4 \times \sqrt{3}}{4} \times a^2 = \sqrt{3}\, a^2$

Total surface area = $a^2 + \sqrt{3}\, a^2$

Activity

A toy is in the shape of a square pyramid of base edge 16 centimetres and slant height 10 centimetres. What is the total cost of painting 500 such toys, at 80 rupees per square metre?

Answer

In a square pyramid shaped toy,

Base edge, a = 16 cm

Slant height, l = 10 cm

Painting area = Total surface area = $a^2 + 2al$

$= 16^2 + 2 \times 16 \times 10 = 256 + 320 = 576\ cm^2$

Area of such 500 toys = $500 \times 576\ cm^2 = 288000\ cm^2$

$= \frac{288000}{10000} = 28.8\ m^2$

[For converting cm^2 into m^2 divide by 10000. $1 m^2 = 10000\ cm^2$]

Therefore, total painting area = $28.8\ m^2$

Cost of painting for $1\ m^2$ = Rs. 80

Total cost of painting at the rate Rs.80/m^2 = 28.8×80 = Rs. 2304

Activity

The lateral faces of a square pyramid are equilateral triangles and the length of a base edge is 30 centimetres. What is its surface area?

Answer

Lateral faces are equilateral triangles.

Base edge, a = 30 cm

Surface area = $a^2 + \sqrt{3}a^2$

$= 30^2 + \sqrt{3} \times 30^2$

$= 900 + 900\sqrt{3}\ \text{cm}^2$

$= 900(1 + \sqrt{3})\ \text{cm}^2$

$= 900(1 + 1.73)\ \text{cm}^2$

$= 900 \times 2.73 = 2457\ \text{cm}^2$

Assignment

The perimeter of the base of square pyramid is 40 centimetres and the total length of all its edges is 92 centimetres. Calculate its surface area.

Prepared by Jaisingh Jose G R ;HST(Maths) Govt.V&HSS Kulathoor

Assignment on 16-12-2020

The perimeter of the base of square pyramid is 40 centimetres and the total length of all its edges is 92 centimetres. Calculate its surface area.

Answer

Base perimeter, $4a = 40$ cm

Therefore, Base edge, $a = \frac{40}{4} = 10$cm

Length of all edges = 92 cm

That is, 4 × base edge + 4 × lateral edge = 92 cm

That is, $4a + 4e = 92$

$40 + 4e = 92$

$4e = 92 - 40 = 52$

$e = \frac{52}{4} = 13$ cm

Slant height, l $= \sqrt{e^2 - (\frac{a}{2})^2}$

$= \sqrt{13^2 - (\frac{10}{2})^2}$

$= \sqrt{169 - 5^2} = \sqrt{169 - 25}$

$= \sqrt{144} = 12$ cm

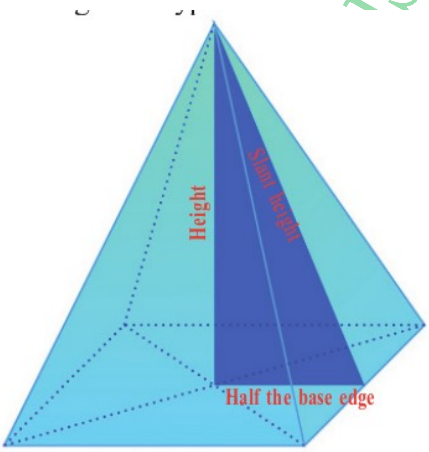
Surface area = $a^2 + 2al$

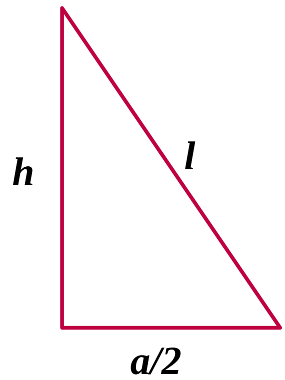
$= 10^2 + 2 \times 10 \times 12 = 100 + 240 = 340\ cm^2$

Note:

Relation between height(h), half of base edge($\frac{a}{2}$) and slant height(l)

Slant height, l $= \sqrt{h^2 + \left(\frac{a}{2}\right)^2}$

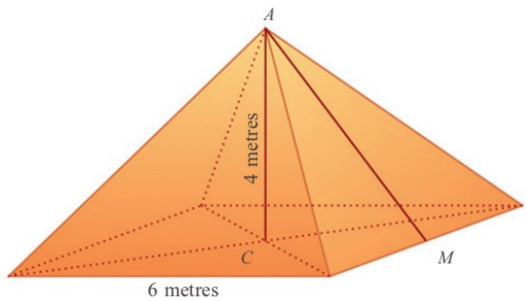




Activity

> A tent is to be made in the shape of a square pyramid of base edges 6 metres and height 4 metres. How many square metres of canvas is needed to make it?

Answer



In a square pyramid shaped tent,

Base edge, a = 6 m

height, h = 4 m

Slant height, l $= \sqrt{h^2 + \left(\frac{a}{2}\right)^2} = \sqrt{4^2 + \left(\frac{6}{2}\right)^2}$

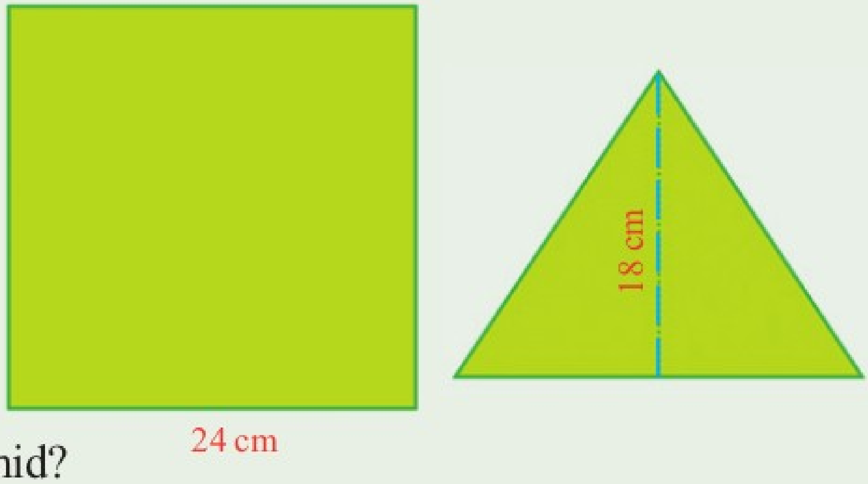
$= \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ **m**

Lateral surface area = $2al = 2 \times 6 \times 5 = 60\ m^2$

Therefore, Area of canvas needed to make the tent = $60\ m^2$

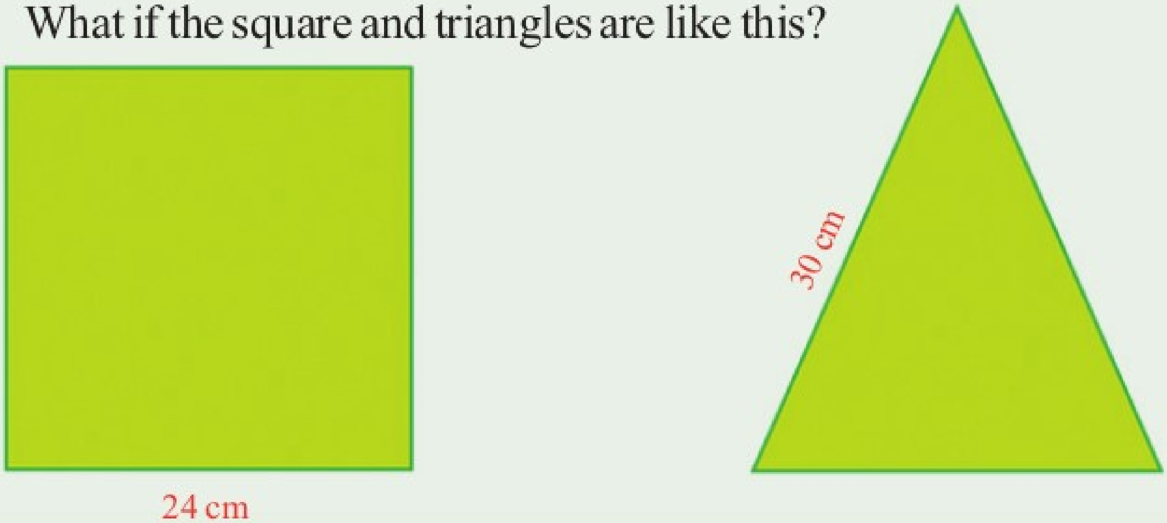
Activity

Using a square and four triangles with dimensions as specified in the picture, a pyramid is made.



What is the height of this pyramid?

What if the square and triangles are like this?



Answer

(a) Base edge, a = 24 cm

Slant height, l = 18 cm

Height, h = $\sqrt{l^2-\left(\frac{a}{2}\right)^2} = \sqrt{18^2-\left(\frac{24}{2}\right)^2}$

$= \sqrt{18^2-12^2} = \sqrt{324-144} = \sqrt{180} = 6\sqrt{5}$ **cm**

(b) Base edge, a = 24 cm

Lateral edge, e = 30 cm

(Slant height)^2, l^2 = $e^2-\left(\frac{a}{2}\right)^2 = 30^2-\left(\frac{24}{2}\right)^2$

$= 30^2-12^2 = 900-144 = 756$

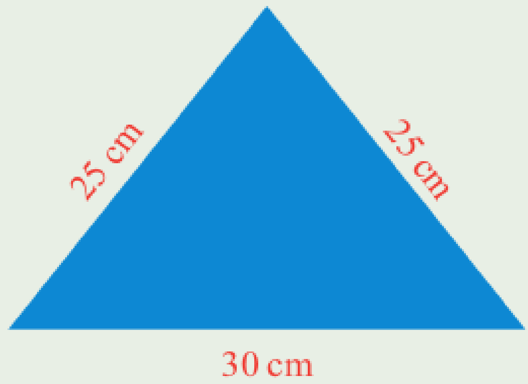
(Height)^2, h^2 = $l^2 - \left(\frac{a}{2}\right)^2 = 756 - \left(\frac{24}{2}\right)^2$

$= 756 - 12^2 = 756 - 144 = 612$

Therefore, Height, h = $\sqrt{612}$

Assignment

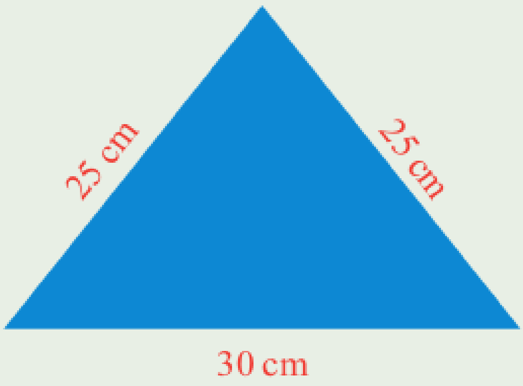
A square pyramid is to be made with the triangle shown here as a lateral face. What would be its height? What if the base edge is 40 centimetres instead of 30 centimetres?



Prepared by Jaisingh Jose G R ;HST(Maths) Govt.V&HSS Kulathoor

Assignment on 18-12-2020

A square pyramid is to be made with the triangle shown here as a lateral face. What would be its height? What if the base edge is 40 centimetres instead of 30 centimetres?



Answer

(a) Base edge, a =30 cm

Lateral edge, e = 25 cm

Slant height, l $= \sqrt{e^2 - \left(\frac{a}{2}\right)^2}$

$$= \sqrt{25^2 - \left(\frac{30}{2}\right)^2}$$

$$= \sqrt{25^2 - 15^2} = \sqrt{625 - 225}$$

$$= \sqrt{400} = 20 \text{ cm}$$

Height, h $= \sqrt{l^2 - \left(\frac{a}{2}\right)^2}$

$$= \sqrt{20^2 - (\frac{30}{2})^2}$$

$$= \sqrt{20^2 - 15^2} = \sqrt{400 - 225}$$

$$= \sqrt{175} = 5\sqrt{7} \text{ cm}$$

(b) If the base edge, a = 40 cm

Slant height, l $= \sqrt{e^2 - (\frac{a}{2})^2}$

$$= \sqrt{25^2 - (\frac{40}{2})^2}$$

$$= \sqrt{25^2 - 20^2} = \sqrt{625 - 400}$$

$$= \sqrt{225} = 15 \text{ cm}$$

The slant height of a square pyramid should always be grater than half of base edge. Here $l < (\frac{a}{2})$

Therefore, A square pyramid with given measure cannot be made.

Activity

Prove that in any square pyramid, the squares of the height, slant height and lateral edge are in arithmetic sequence.

Answer

We have to prove h², l², e² are in arithmetic sequence.

In a square pyramid; if the height is h, half of base edge is $(\frac{a}{2})$ **and slant height is l, then**

$$h^2 = l^2 - (\frac{a}{2})^2$$

$$e^2 = l^2 + (\frac{a}{2})^2$$

$l^2-\left(\frac{a}{2}\right)^2$, l^2, $l^2-\left(\frac{a}{2}\right)^2$ are in arithmetic sequence with common difference $\left(\frac{a}{2}\right)^2$.

Therefore,

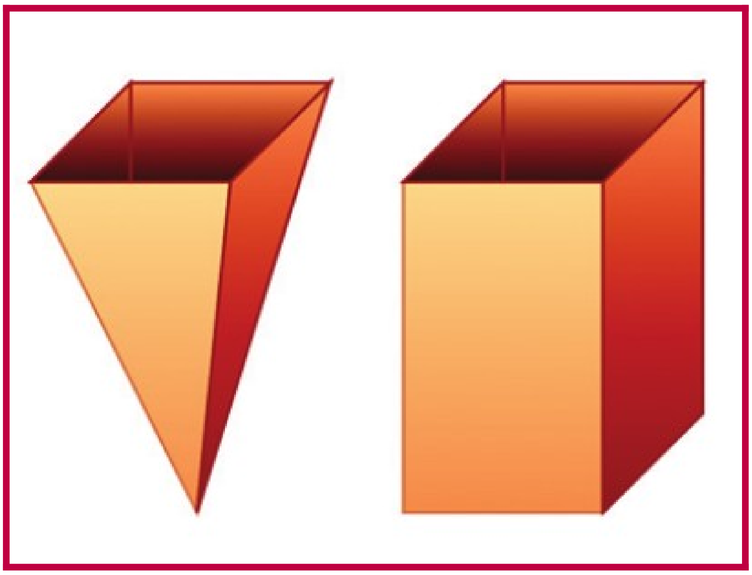
h^2, l^2, e^2 are in arithmetic sequence with common difference $\left(\frac{a}{2}\right)^2$

That is, squares of height, slant height and lateral edge are in arithmetic sequence.

Volume of a pyramid

The volume of any prism is equal to the product of the base area and the height. That is Volume of square prism, V = $a^2 \times h$

Make a hollow square pyramid with thick paper and also a square prism of the same base and height. Fill the pyramid with sand and transfer it to the prism. Repeat this process 3 times. Now we can see the square prism is completely filled with sand. Thus we see that the volume of the prism is three times the volume of the pyramid. Other wise, the volume of the pyramid is one third $\left(\frac{1}{3}\right)$ of the volume of the prism.



Therefore,

Volume of the square pyramid, V = $\frac{1}{3} \times base\, area \times height$

$= \frac{1}{3} a^2 \times h$

VOLUME OF A SQUARE PYRAMID IS ONE THIRD OF THE VOLUME OF A SQUARE PRISM WITH SAME BASE AREA AND HEIGHT.

Discussed in previous class

Volume of the square pyramid, V = $\frac{1}{3} \times basearea \times height$

$$= \frac{1}{3} a^2 \times h$$

Activity

What is the volume of a square pyramid of base edge 10 centimetres and slant height 15 centimetres?

Answer

(a) Base edge, a = 10 cm

Slant height, l = 15 cm

Height of the square pyramid, h $= \sqrt{l^2 - \left(\frac{a}{2}\right)^2}$

$$= \sqrt{15^2 - \left(\frac{10}{2}\right)^2}$$

$$= \sqrt{15^2 - 5^2}$$

$$= \sqrt{225 - 25}$$

$$= \sqrt{200} = 10\sqrt{2} \text{ cm}$$

Volume of the square pyramid, V = $\frac{1}{3} \times basearea \times height$

$= \frac{1}{3} a^2 \times h$

$= \frac{1}{3} \times 10^2 \times 10\sqrt{2}$

$= \frac{1000\sqrt{2}}{3}$ cm^3

Activity

Two square pyramids have the same volume. The base edge of one is half that of the other. How many times the height of the second pyramid is the height of the first?

Answer

Let, Base edge of the first square pyramid = a_1

Height of the first square pyramid = h_1

Base edge of the second square pyramid = a_2

Height of the second square pyramid = h_2

$a_2 = \frac{a_1}{2}$ (Given)

Volume of the first square pyramid, $V_1 = \frac{1}{3} a_1^2 \times h_1$

Volume of the second square pyramid, $V_2 = \frac{1}{3} a_2^2 \times h_2$

Volume of two square pyramids are same.

That is, $V_1 = V_2$

That is, $\frac{1}{3}a_1^2 \times h_1 = \frac{1}{3}a_2^2 \times h_2$

That is, $\frac{1}{3}a_1^2 \times h_1 = \frac{1}{3} \times \left(\frac{a_1}{2}\right)^2 \times h_2$

That is, $\frac{1}{3}a_1^2 \times h_1 = \frac{1}{3} \times \frac{a_1^2}{4} \times h_2$

That is, $h_1 = \frac{h_2}{4}$

That is, $h_2 = 4h_1$

That is, Height of the second square pyramid is 4 times the height of the first square pyramid.

Activity

The base edges of two square pyramids are in the ratio 1 : 2 and their heights in the ratio 1 : 3. The volume of the first is 180 cubic centimetres. What is the volume of the second?

Answer

Let, Base edge of the first square pyramid = a_1

Height of the first square pyramid = h_1

Base edge of the second square pyramid = a_2

Height of the second square pyramid = h_2

$a_1 : a_2 = 1:2$

Therefore, if $a_1 = a$ then $a_2 = 2a$

$h_1 : h_2 = 1:3$

Therefore, if $h_1 = h$ then $h_2 = 3h$

Volume of the first square pyramid, V_1 = $\frac{1}{3}a_1^2 \times h_1$

$= \frac{1}{3}a^2 \times h$

Volume of the second square pyramid, V_2 = $\frac{1}{3}a_2^2 \times h_2$

$= \frac{1}{3}(2a)^2 \times 3h$

$= \frac{1}{3} \times 4a^2 \times 3h$

Therefore,

Ratio of volumes = $\frac{1}{3}a^2 \times h : \frac{1}{3} \times 4a^2 \times 3h$

$= \frac{1}{3}a^2 \times h : \frac{1}{3} \times 4a^2 \times 3h$

= 1:12

Volume of the second square pyramid = 180

Therefore, Volume of the second square pyramid = 12 × 180

= 2160 cm^3

Activity

The slant height of a square pyramid is 25 centimetres and its surface area is 896 square centimetres. What is its volume?

Answer

Slant height, l = 25 cm

Surface area = 896 cm^2

That is, a^2+2al = 896

$a^2+2a\times25$ = 896

a^2+50a = 896

For completing the square add 25^2 on both sides.

That is, $a^2+50a+25^2$ = 896 + 25^2

That is, $(a+25)^2$ = 896 + 625

That is, $(a+25)^2$ = 1521

That is, $(a+25)^2$ = 39^2

Therefore, $(a+25)$ = 39

a = 39 – 25 = 14 cm

Height of the square pyramid, h $= \sqrt{l^2-(\frac{a}{2})^2}$

$= \sqrt{25^2-(\frac{14}{2})^2}$

$= \sqrt{25^2-7^2}$

$= \sqrt{625-49}$

$= \sqrt{576}$ = 24 cm

Volume of the square pyramid, V = $\frac{1}{3}\times basearea\times height$

$= \frac{1}{3}a^2\times h$

$$= \frac{1}{3} \times 14^2 \times 24$$

$$= 14^2 \times 8$$

$$= 196 \times 8 = \mathbf{1568\ cm^3}$$

Assignment

All edges of a square pyramid are 18 centimetres. What is its volume?

Prepared by Jaisingh Jose G R ;HST(Maths) Govt.V&HSS Kulathoor

Assignment on 21-12-2020

All edges of a square pyramid are 18 centimetres. What is its volume?

Answer

Base edge, a = 18 cm

Lateral edge, e = 18 cm

Slant height of the square pyramid, l $= \sqrt{e^2-\left(\frac{a}{2}\right)^2}$

$$= \sqrt{18^2-\left(\frac{18}{2}\right)^2}$$

$$= \sqrt{18^2-9^2}$$

$$= \sqrt{324-81}$$

$$= \sqrt{243}$$

$$= \sqrt{81\times 3}$$

$$= 9\sqrt{3}$$

Height of the square pyramid, h $= \sqrt{l^2-\left(\frac{a}{2}\right)^2}$

$$= \sqrt{(9\sqrt{3})^2-\left(\frac{18}{2}\right)^2}$$

$$= \sqrt{243-9^2}$$

$$= \sqrt{243-81}$$

$$= \sqrt{162}$$
$$= \sqrt{81 \times 2}$$
$$= 9\sqrt{2}$$

Volume of the square pyramid, V = $\frac{1}{3} \times base\ area \times height$

$$= \frac{1}{3} \times a^2 \times h$$

$$= \frac{1}{3} \times 18^2 \times 9\sqrt{2}$$

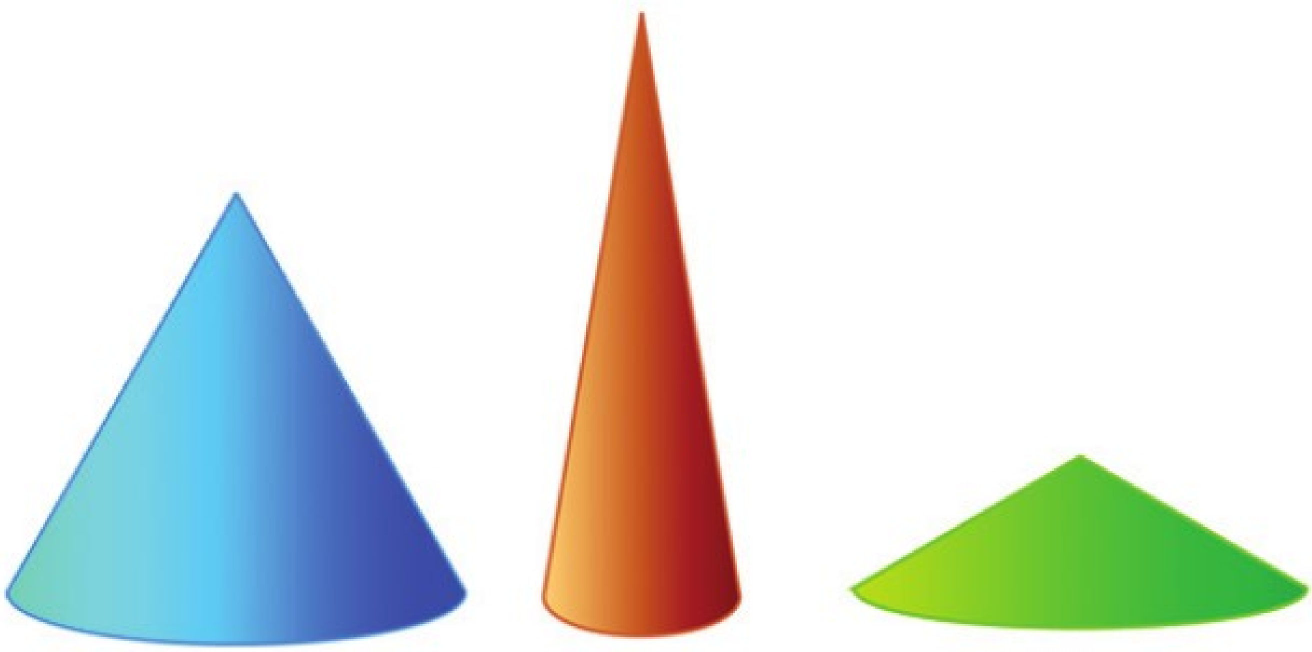
$$= 18^2 \times 3\sqrt{2}$$

$$= 324 \times 3\sqrt{2}$$

$$= 972\sqrt{2}\ \text{cm}^3$$

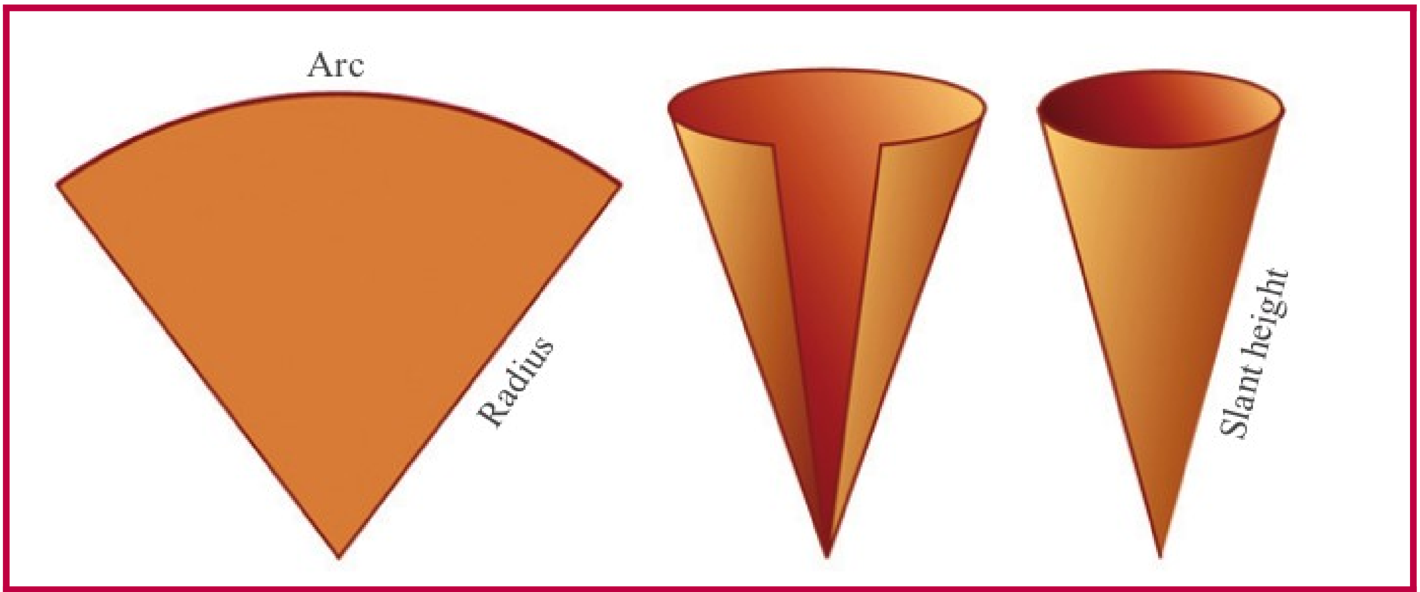
Cone

Cylinders are prism-like solids with circular bases. Similarly, Pyramid-like solids with circular bases are called cones.



we can make a cone by rolling up a sector of a circle.

What is the relation between the dimensions of the sector and the cone made from it?

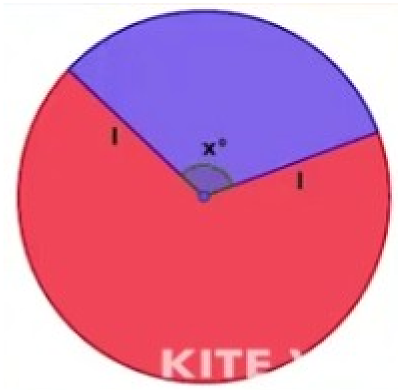


The radius of the sector becomes the slant height of the cone. The arc length of the sector becomes the circumference of the base of the cone.

Let, The radius of the sector = l

Central angle of the sector = x and

Radius of the cone made from it = r



Then, Arc length of the sector = $2\pi l \times \frac{x}{360}$

Perimeter or circumference of base of the cone = $2\pi r$

These two measures are equal.

Therefore, $2\pi r = 2\pi l \times \frac{x}{360}$

That is, $r = l \times \frac{x}{360}$

That is, $\frac{r}{l} = \frac{x}{360}$

Therefore, $r : l = x : 360$

Activity

What are the radius of the base and slant height of a cone made by rolling up a sector of central angle 60° cut out from a circle of radius 10 centimetres?

Answer

Radius of the sector = 10 cm

Central angle of the sector, x = 60°

Therefore,

Slant height of the cone, l = 10 cm

Base radius of the cone made from it, r = $l \times \frac{x}{360}$

$$= 10 \times \frac{60°}{360} = \frac{60°}{36} = \frac{5}{3} \text{ cm}$$

Activity

What is the central angle of the sector to be used to make a cone of base radius 10 centimetres and slant height 25 centimetres?

Answer

Slant height of the cone, l = 25 cm

Base radius of the cone, r = 10 cm

Let, Central angle of the sector = x

Base radius of the cone, r = $l \times \frac{x}{360}$

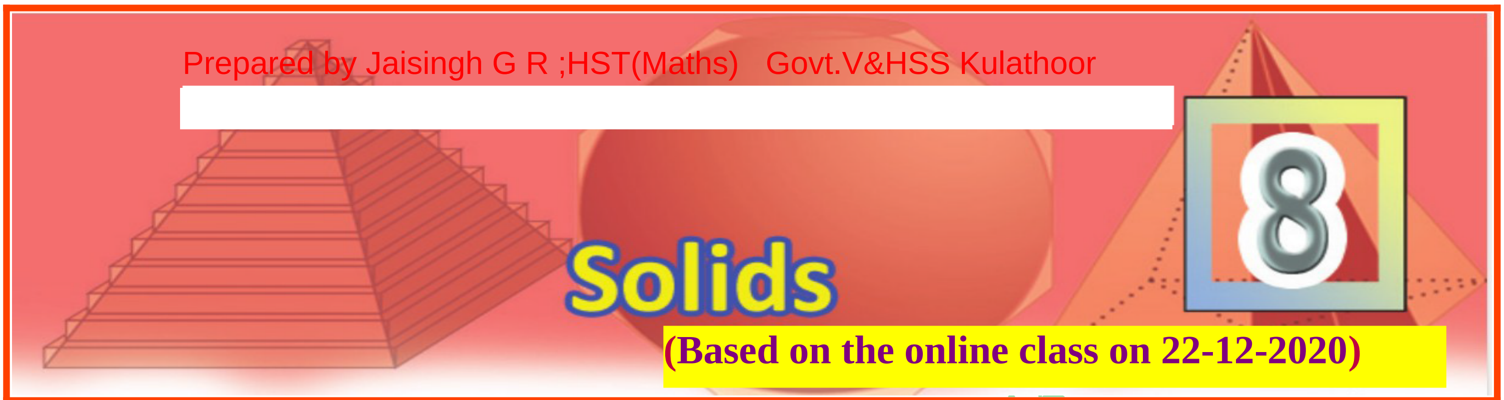
That is, $10 = 25 \times \frac{x}{360}$

$$25 \times x = 360 \times 10$$

$$x = \frac{360 \times 10}{25} = 144°$$

Assignment

What is the ratio of the base-radius and slant height of a cone made by rolling up a semicircle?



Assignment on 22-12-2020

What is the ratio of the base-radius and slant height of a cone made by rolling up a semicircle?

Answer

If, The radius of the sector = l

Central angle of the sector = x and

Radius of the cone made from it = r, then

r : l = x : 360

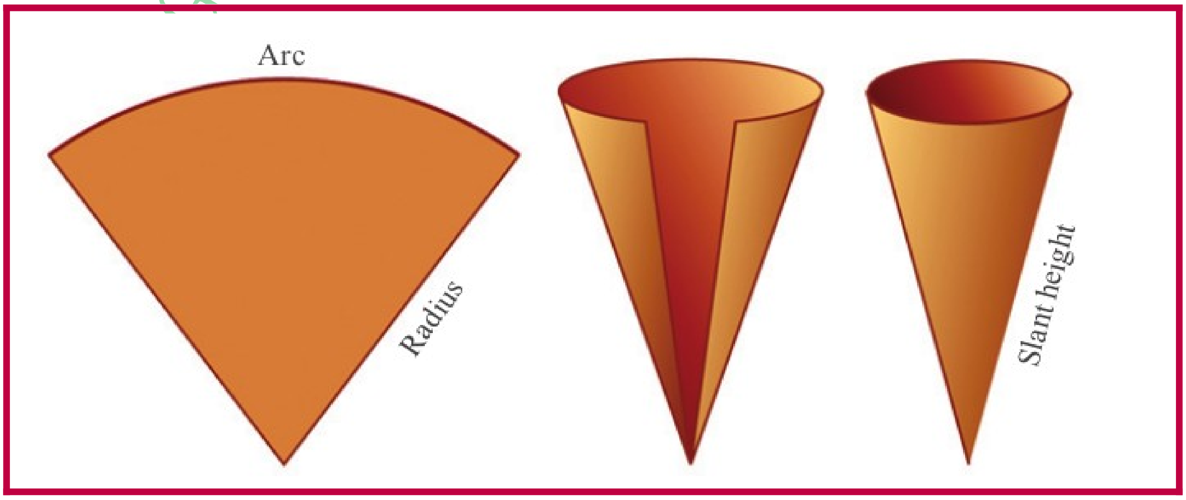
Here, x = central angle of a semicircle = 180°

Therefore,

Ratio of base radius and slant height = 180 : 360 = 1 : 2

Curved surface area of a cone

A cone also has a curved surface. The area of this curved surface is the area of the sector used to make the cone.



If, The radius of the sector = l

Central angle of the sector = x and

Radius of the cone made from it = r, then

Area of the sector = $\pi l^2 \times \frac{x}{360}$

{ we know that $\frac{x}{360} = \frac{r}{l}$ }

Therefore, Area of the sector = $\pi l^2 \times \frac{r}{l} = \pi rl$

Therefore, Curved surface area of a cone = πrl

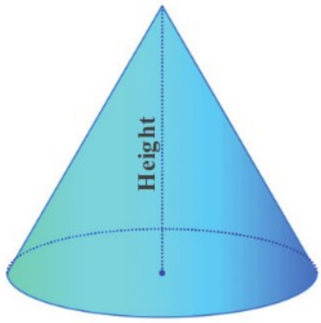
That is, Curved surface area of a cone is "half of the product of its base perimeter and slant height'.

Total surface area of a cone

Total Surface area of a cone = Base area + Curved surface area

$= \pi r^2 + \pi rl$

Height of a cone



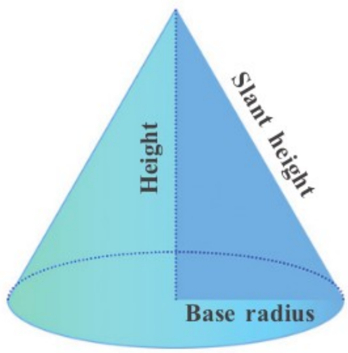
Height of a cone is distance between the apex and the centre of the base.

Relation between base radius(r), height(h) and slant height(l) of a cone

$$(slant\,height)^2 = (height)^2 + (base\,radius)^2$$

That is, $l^2 = h^2 + r^2$ or

$$l = \sqrt{h^2 + r^2}$$



Activity

What is the area of the curved surface of a cone of base radius 12 centimetres and slant height 25 centimetres?

Answer

Base radius, r = 12 cm

Slant height, l = 25 cm

Area of curved surface = $\pi rl = \pi \times 12 \times 25 = 300\pi\ cm^2$

Activity

What is the surface area of a cone of base diameter 30 centimetres and height 40 centimetres?

Answer

Base diameter, d = 30 cm

Therefore, Base radius, r = 15 cm

Height, h = 40 cm

We know that, $(slant\ height)^2$, $l^2 = h^2 + r^2$

$$l^2 = 40^2 + 15^2$$
$$= 1600 + 225$$
$$= 1825$$
$$l = \sqrt{1825}$$

Surface area = Base area + Curved surface area

$$= \pi r^2 + \pi rl$$
$$= \pi \times 15 \times 15 + \pi \times 15 \times \sqrt{1825}$$
$$= 225\pi + \pi \times 15 \times \sqrt{5 \times 5 \times 73}$$
$$= 225\pi + \pi \times 15 \times 5\sqrt{73}$$
$$= 225\pi + 75\sqrt{73}\pi\ cm^2$$

Assignment

Prove that for a cone made by rolling up a semicircle, the area of the curved surface is twice the base area.

Assignment on 23-12-2020

> Prove that for a cone made by rolling up a semicircle, the area of the curved surface is twice the base area.

Answer

For a cone made by rolling up a semicircle,

central angle of sector, x = 180°

Let, radius of the cone = r

slant height = l

We know that, $\frac{r}{l} = \frac{x}{360}$

$$\frac{r}{l} = \frac{180}{360}$$

That is, $\frac{r}{l} = \frac{1}{2}$

Therefore, $1 \times l = 2 \times r$

That is, $l = 2r$

Base area of the cone = πr^2

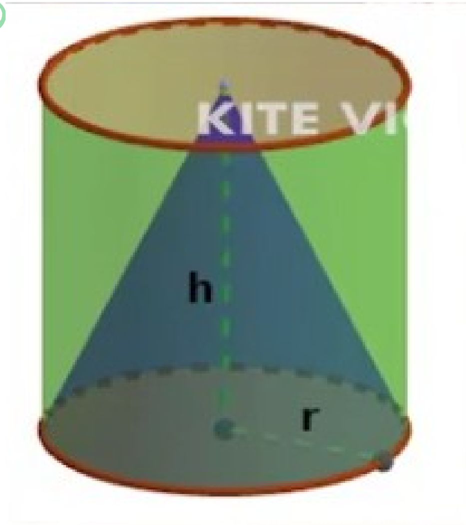
Curved surface area of the cone = $\pi r l = \pi r \times 2r = 2\pi r^2$

That is, $curved surface area of the cone = 2 \times base area of the cone$

That is, For a cone made by rolling up a semicircle, area of curved surface is twice the base area.

Volume of a cone

Make a cone and a cylinder of the same base and height. Fill the cone with sand and transfer it to the cylinder. Repeat this process three times. Now we can see that the cylinder is completely filled with sand.



Therefore, The volume of the cone is a third of the volume of the cylinder.

Volume of a cone $= \frac{1}{3} \times \textit{volume of a cylinder of same dimensions}$

$$= \frac{1}{3} \times basearea \times height$$

$$= \frac{1}{3} \times \pi r^2 \times h$$

The volume of a cone is equal to a third of the product of the base area and height.

Activity

The base radius and height of a cylindrical block of wood are 15 centimetres and 40 centimetres. What is the volume of the largest cone that can be carved out of this?

Answer Radius of the cylinder = 15 cm

Height of the cylinder = 40 cm

Therefore,

Radius of the largest cone that can be carved out from the cylinder, r = 15 cm

Height of the largest cone that can be carved out from the cylinder, h = 40 cm

Volume of the cone, V = $\frac{1}{3} \times basearea \times height$

$$= \frac{1}{3} \times \pi \times r^2 \times h$$

$$= \frac{1}{3} \times \pi \times 15^2 \times 40$$

$$= \frac{1}{3} \times \pi \times 225 \times 40$$

$$= \pi \times 75 \times 40$$

$$= 3000\pi \text{ cm}^3$$

Activity

The base radius and height of a solid metal cylinder are 12 centimetres and 20 centimetres. By melting it and recasting, how many cones of base radius 4 centimetres and height 5 centimetres can be made?

Answer

Radius of the cylinder, R = 12 cm

Height of the cylinder, H = 20 cm

Volume of the cylinder, V $= \pi \times R^2 \times H$

$= \pi \times 12^2 \times 20$

$= \pi \times 12 \times 12 \times 20$ **cm³**

Radius of one cone, r = 4 cm

Height of one cone, h = 5 cm

Volume of one cone, v $= \frac{1}{3} \times \pi \times r^2 \times h$

$= \frac{1}{3} \times \pi \times 4^2 \times 5$

$= \frac{1}{3} \times \pi \times 4 \times 4 \times 5$ **cm³**

Volume of the cylinder = Volume of cones made from it

Therefore, Number of cones that can be made by melting the cylinder = $\dfrac{Volume of the cylinder}{Volume of one cone}$

$$= \frac{\pi \times 12 \times 12 \times 20}{\frac{1}{3} \times \pi \times 4 \times 4 \times 5}$$

$$= \frac{3 \times \pi \times 12 \times 12 \times 20}{1 \times \pi \times 4 \times 4 \times 5}$$

$= 3 \times 3 \times 3 \times 4$ **= 108**

Activity

A sector of central angle 216° is cut out from a circle of radius 25 centimetres and is rolled up into a cone. What are the base radius and height of the cone? What is its volume?

Answer

Central angle of the sector, x = 216°

Radius of the sector = Slant height of the cone, l = 25 cm

Let, radius of the cone made from this sector = r and height = h

We know that, r = $\frac{l \times x}{360} = \frac{25 \times 216}{360}$

$= \frac{25 \times 6}{10} = \frac{5 \times 6}{2} = 5 \times 3$ **= 15 cm**

Height of the cone, h = $\sqrt{l^2 - r^2} = \sqrt{25^2 - 15^2}$

$= \sqrt{625 - 225} = \sqrt{400}$ **= 20 cm**

Volume of the cone, v = $\frac{1}{3} \times \pi \times r^2 \times h$

$$= \frac{1}{3} \times \pi \times 15^2 \times 20$$

$$= \frac{1}{3} \times \pi \times 225 \times 20$$

$$= \frac{1}{3} \times \pi \times 225 \times 20$$

$$= \pi \times 75 \times 20 = 1500\pi \text{ cm}^3$$

Activity

Two cones have the same volume and their base radii are in the ratio 4 : 5. What is the ratio of their heights?

Answer

Ratio of radii = 4 : 5

Therefore, radius of the first cone = 4k and

radius of the second cone = 5k

Let, height of the first cone = h_1

height of the second cone = h_2

Volume of the first cone = $\frac{1}{3} \times \pi \times (4k)^2 \times h_1$

Volume of the second cone = $\frac{1}{3} \times \pi \times (5k)^2 \times h_2$

Since volumes of both cones are equal,

$$\frac{1}{3} \times \pi \times (4k)^2 \times h_1 = \frac{1}{3} \times \pi \times (5k)^2 \times h_2$$

$$(4k)^2 \times h_1 = (5k)^2 \times h_2$$

$$16k^2 \times h_1 = 25k^2 \times h_2$$

$$16 \times h_1 = 25 \times h_2$$

Therefore, $\frac{h_1}{h_2} = \frac{25}{16}$

Therefore, $h_1 : h_2 = 25:16$

That is, ratio of heights = $25:16$

Assignment

The base radii of two cones are in the ratio 3 : 5 and their heights are in the ratio 2 : 3. What is the ratio of their volumes?

Prepared by Jaisingh G R ;HST(Maths) Govt.V&HSS Kulathoor

(Based on the online class on 29-12-2020)

Assignment on 28-12-2020

The base radii of two cones are in the ratio 3 : 5 and their heights are in the ratio 2 : 3. What is the ratio of their volumes?

Answer

Ratio of radii, = 3 : 5

Therefore, radius of first cone = 3r

radius of second cone = 5r

Ratio of heights = 2 : 3

Therefore, height of first cone = 2h

height of second cone = 3h

volume of the first cone $= \frac{1}{3} \times base\ area \times height$

$$= \frac{1}{3} \times \pi \times (3r)^2 \times 2h$$

volume of the second cone $= \frac{1}{3} \times base\ area \times height$

$$= \frac{1}{3} \times \pi \times (5r)^2 \times 3h$$

Ratio of volumes $= \frac{1}{3}\times\pi\times(3r)^2\times2h : \frac{1}{3}\times\pi\times(5r)^2\times3h$

$= (3r)^2\times2h : (5r)^2\times3h$

$= 9r^2\times2h : 25r^2\times3h$

$= 9\times2 : 25\times3$

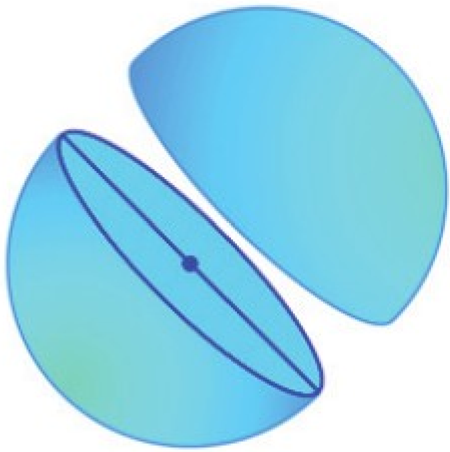
= 18 : 75 = 6 : 25

Sphere

A sphere has only one face.

If we slice a sphere, we get a circle.

A sphere also has a centre, from which the distance to any point on its surface is the same. This distance is called the radius of the sphere and double this is called the diameter.



If we slice a sphere into exact halves, we get a circle whose centre, radius and diameter are those of the sphere itself.

Surface area of a sphere

We cannot make the surface of a sphere flat without some folding or stretching. But we can prove that the surface area of a sphere of radius r is $4\pi r^2$

The surface area of a sphere is equal to the square of its radius multiplied by 4π.

Volume of a sphere

We can prove that the volume of a sphere of radius r = $\frac{4}{3} \times \pi r^3$

Hemisphere

If we slice a sphere into exact halves, we get two hemispheres.

A hemisphere has two faces. One flat face and one curved face.

Surface area of a hemisphere

If the radius of a hemisphere is r,

Area of flat surface = πr^2

Area of curved surface = $2\pi r^2$

Therefore,

Total surface area of a hemisphere = $\pi r^2 + 2\pi r^2 = 3\pi r^2$

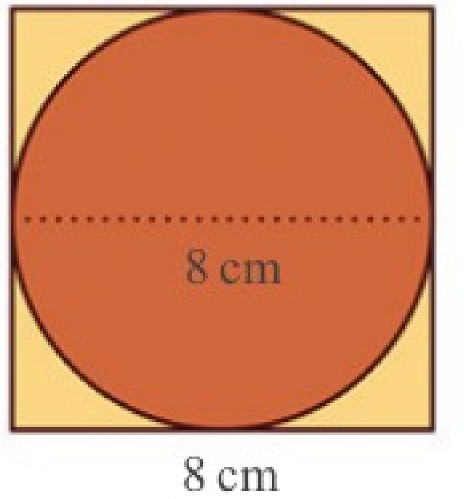
Volume of a hemisphere

Volume of a hemisphere of radius r = $\frac{2}{3} \times \pi r^3$

Activity

What is the surface area of the largest sphere that can be carved from a cube of edges 8 centimetres?

Answer



Length of side of the cube = Diameter of largest sphere that can be carved out from the cube = 8 cm

Therefore, radius r = 4 cm

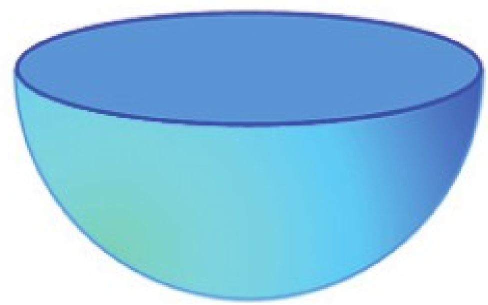
Surface area of the sphere $= 4\pi r^2$

$= 4 \times \pi \times 4^2$

$= 64\pi \text{ cm}^2$

Activity

A solid sphere of radius 12 centimetres is cut into two equal halves. What is the surface area of each hemisphere?



Answer

Radius of the hemisphere r = 12 cm

Surface area of the hemisphere $= 3\pi r^2$

$= 3 \times \pi \times 12^2$

$= 3 \times \pi \times 144$

$= 432\pi \text{ cm}^2$

Activity

The surface area of a solid sphere is 120 square centimetres. If it is cut into two halves, what would be the surface area of each hemisphere?

Answer

Surface area of a solid sphere = 120 cm^2

Therefore, $4\pi r^2 = 120 \text{ cm}^2$

Therefore, $\pi r^2 = \frac{120}{4} = 30 \text{ cm}^2$

Therefore,

Surface area of each hemisphere = $3\pi r^2 = 3 \times 30 = 90 \text{ cm}^2$

Activity

The volumes of two spheres are in the ratio 27 : 64. What is the ratio of their radii? And the ratio of their surface areas?

Answer

Let, radius of the first sphere = r_1

radius of the second sphere = r_2

volume of the first sphere = $v_1 = \frac{4}{3} \times \pi (r_1)^3$

volume of the second sphere = $v_2 = \frac{4}{3} \times \pi (r_2)^3$

$v_1 : v_2 = 27 : 64$

That is, $\frac{4}{3}\times\pi(r_1)^3 : \frac{4}{3}\times\pi(r_2)^3 = 27 : 64$

Therefore, $(r_1)^3 : (r_2)^3 = 27 : 64$

$(r_1)^3 : (r_2)^3 = 3^3 : 4^3$

Therefore, ratio of radii = $r_1 : r_2 = 3 : 4$

Therefore, radius of the first sphere = 3k

radius of the second sphere = 4k

surface area of the first sphere = $4\pi(r_1)^2 = 4\pi(3k)^2$

surface area of the second sphere = $4\pi(r_2)^2 = 4\pi(4k)^2$

Ratio of surface area = $4\pi(3k)^2 : 4\pi(4k)^2$

$= (3k)^2 : (4k)^2$

$= 9k^2 : 16k^2$

$= 9 : 16$

Assignment

If the ratio of the radii of two spheres is 2:3, find the ratio of their Volumes.

Assignment on 29-12-2020

If the ratio of the radii of two spheres is 2:3, find the ratio of their Volumes.

Answer

Ratio of radii of two spheres = 2 : 3

Therefore, radius of first sphere = 2r

radius of second sphere = 3r

volume of the first sphere $= \frac{4}{3}\pi(2r)^3$

volume of the second sphere = $\frac{4}{3}\pi(3r)^3$

Ratio of volumes $= \frac{4}{3}\pi(2r)^3 : \frac{4}{3}\pi(3r)^3$

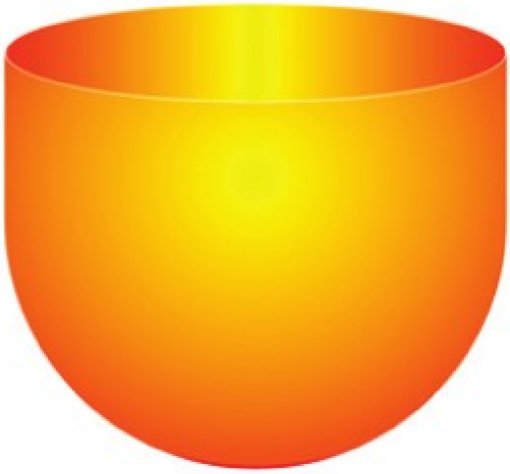
$= (2r)^3 : (3r)^3$

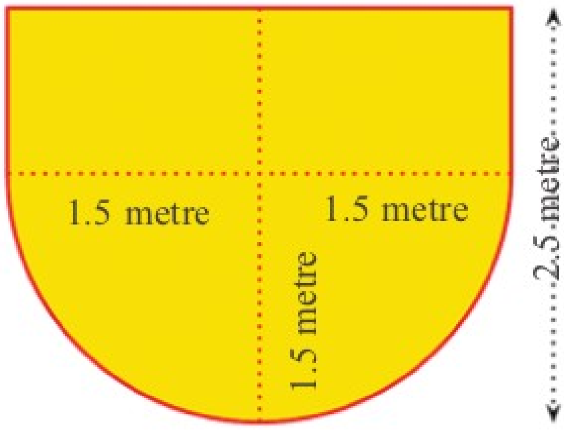
$= 8r^3 : 27r^3$

= 8 : 27

Activity

A water tank is in the shape of a hemisphere attached to a cylinder. Its radius is 1.5 metres and the total height is 2.5 metres. How many litres of water can it hold?





Answer

This shape can be split into one hemisphere and one cylinder.

In the hemisphere,

radius, r = 1.5 m

volume, v = $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 1.5^3$

$= \frac{2}{3}\pi \times 1.5 \times 1.5 \times 1.5$

$= 2\pi \times 0.5 \times 1.5 \times 1.5$

$= \pi \times 1.5 \times 1.5$

$= 2.25\pi \text{ m}^3$

In the cylinder,

radius, r = 1.5 m

height, h = 2.5 – 1.5 = 1 m

volume, v = $\pi r^2 h = \pi \times 1.5^2 \times 1$

$= 2.25\pi$ **m³**

volume of the water tank = volume of hemisphere + volume of cylinder = $2.25\pi + 2.25\pi = 4.5\pi$ **m³**

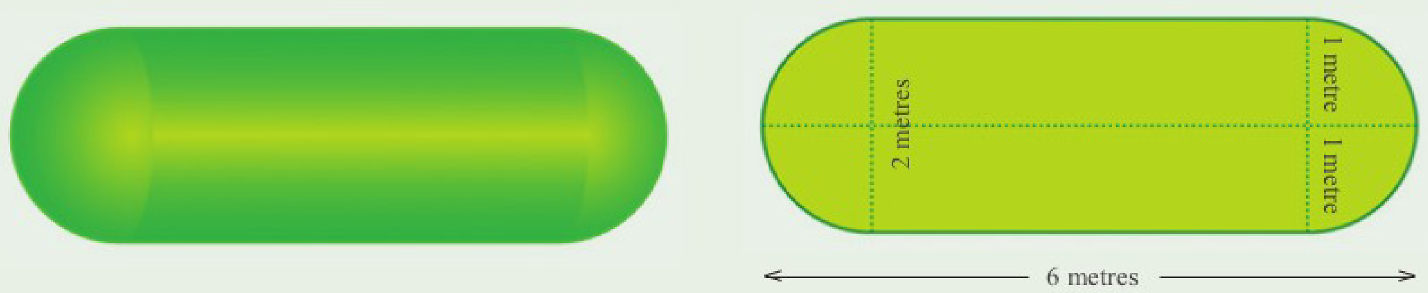
$= 4.5 \times \pi \times 1000$ **ltr {1 m³ = 1000 litre}**

= 14130 ltr

That is, 14130 litres of water can be stored in the tank.

Activity

The picture shows the dimensions of a petrol tank.



How many litres of petrol can it hold?

Answer

This shape can be split into two hemispheres and one cylinder.

In one hemisphere,

radius, r = 1 m

volume, v = $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 1^3 = \frac{2}{3}\pi$ **m³**

In the cylinder,

radius, r = 1 m

height, h = 6 – (1 + 1) = 6 – 2 = 4 m

volume, v = $\pi r^2 h = \pi \times 1^2 \times 4$

$= 4\pi$ m³

Therefore,

volume of the petrol tank = volume of 2 hemispheres + volume of cylinder = $2 \times \frac{2}{3}\pi + 4\pi = \frac{4}{3}\pi + 4\pi$ m³

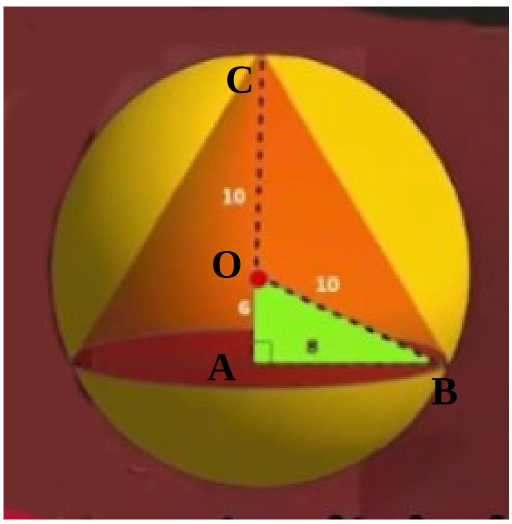
$= \frac{16}{3}\pi$ m³

$= \frac{16000}{3}\pi$ ltr {1 m³ = 1000 litre}

Activity

From a solid sphere of radius 10 centimetres, a cone of height 16 centimetres is carved out. What fraction of the volume of the sphere is the volume of the cone?

Answer



In the sphere,

radius, r = 10 cm

volume, v $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 10^3$

$= \frac{4}{3}\pi \times 10 \times 10 \times 10$ cm³

In the cone,

height, h = 16 cm

In ΔOAB,

OA = AC – OC = 16 – 10 = 6 cm

OB = 10 cm (radius of the sphere)

AB = $\sqrt{OB^2 - OA^2} = \sqrt{10^2 - 6^2} = \sqrt{100 - 36}$

$= \sqrt{64}$ **= 8 cm**

That is, radius of the cone, r = 8 cm

volume, v $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 8^2 \times 16$ **cm³**

Ratio of volumes of cone and sphere $= \dfrac{\frac{1}{3}\pi \times 8^2 \times 16}{\frac{4}{3}\pi \times 10 \times 10 \times 10}$

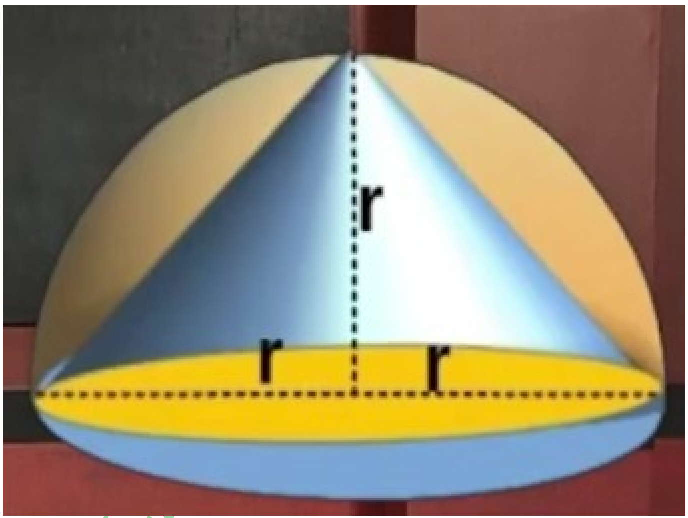
$= \dfrac{8^2 \times 16}{4 \times 10 \times 10 \times 10}$

$= \dfrac{32}{125}$ **= 32 : 125**

Activity

A solid sphere is cut into two hemispheres. From one, a square pyramid and from the other a cone, each of maximum possible size are carved out. What is the ratio of their volumes?

Answer



Let, radius of the sphere = r

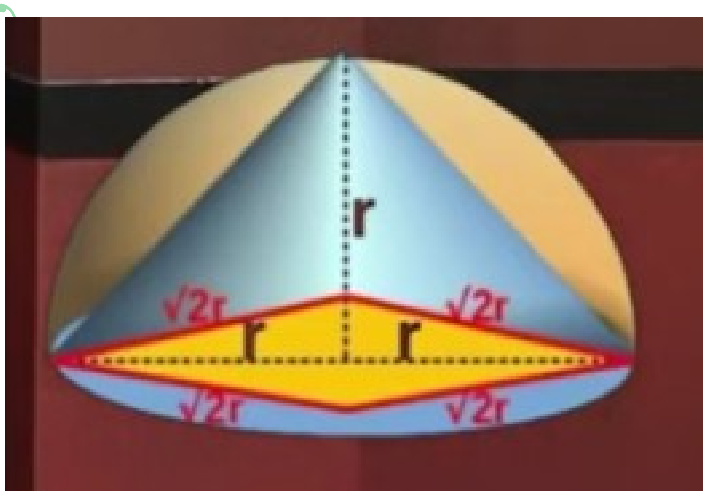
Therefore, radius of the cone made from one hemisphere = r

height of the cone = r

volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times r^2 \times r$

$= \frac{1}{3}\pi \times r^3$ cm^3



Base diagonal of the square pyramid made from other hemisphere = Diameter of the hemisphere = 2r

Therefore, base edge of square pyramid, a = $\frac{2r}{\sqrt{2}} = \sqrt{2}r$

height of the square pyramid, h = r

volume of the cone $= \frac{1}{3}a^2 h$

$= \frac{1}{3}(\sqrt{2}r)^2 \times r$

$= \frac{1}{3} \times 2r^2 \times r = \frac{2}{3}r^3$ cm^3

Ratio of volumes of square pyramid and cone $= \dfrac{\frac{2}{3}r^3}{\frac{1}{3}\pi\times r^3}$

$$= \frac{2}{\pi} = 2:\pi$$

Activity

The base radius and length of a metal cylinder are 4 centimetres and 10 centimetres. If it is melted and recast into spheres of radius 2 centimetres each, how many spheres can be made?

Answer

In the metal cylinder,

`radius, r = 4 cm

height, h = 10 cm

volume = $\pi r^2 h = \pi\times 4^2\times 10$

$= \pi\times 4\times 4\times 10$ **cm³**

In one sphere,

radius, r = 2 cm

volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\times 2^3$

$= \frac{4}{-}\pi\times 2\times 2\times 2$ **cm³**

Therefore,

Number of spheres that can be made by melting the metalic cylinder = $\dfrac{\textit{volume of cylinder}}{\textit{volume of one sphere}}$

$$= \frac{\pi \times 4 \times 4 \times 10}{\frac{4}{3}\pi \times 2 \times 2 \times 2}$$

$$= \frac{3\pi \times 4 \times 4 \times 10}{4\pi \times 2 \times 2 \times 2}$$

$= 3 \times 5$ **= 15**

That is, 15 small spheres of radius 2 cm can be made by melting a metal cylinder of radius 4 cm and height 10 cm.

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