Based on the first bell class on 31-12-2020

Geometry and Algebra 9

Previous knowledge

1. Coordinates of origin is (0,0)
2. The y coordinate of any point on the x axis is 0.
3. The x coordinate of any point on the y axis is 0.
4. The y coordinates of any point in a line parallel to x axis are equal.
5. The x coordinates of any point in a line parallel to y axis are equal.

The distance between the points with coordinates (x_1, y) and (x_2, y) is $|x_1 - x_2|$.

The distance between the points with coordinates (x, y_1) and (x, y_2) is $|y_1 - y_2|$.

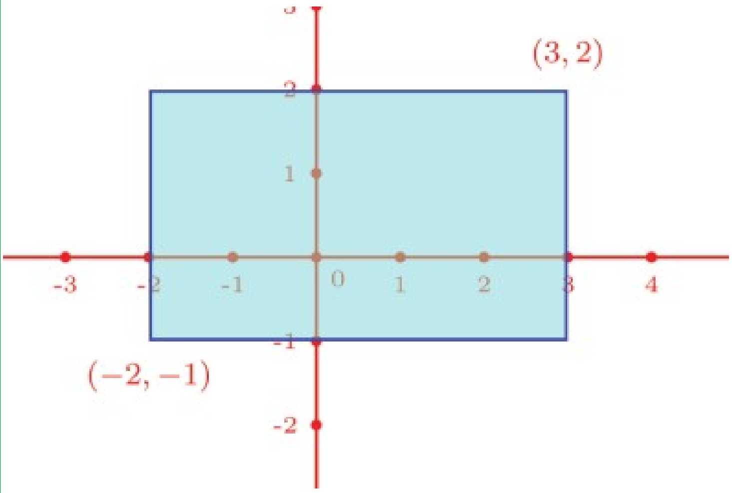
The distance between the point with coordinates (x, y) and the origin is

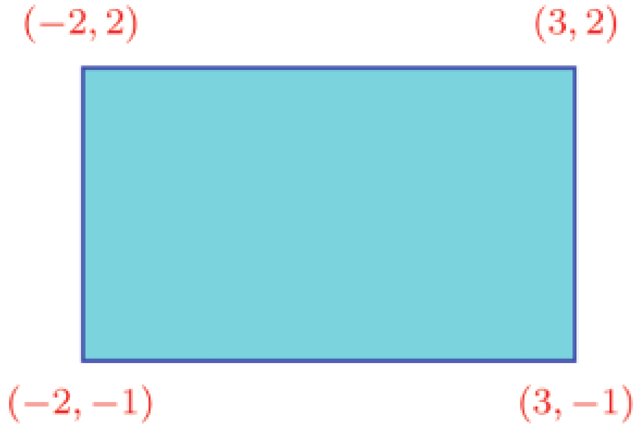
$$\sqrt{x^2 + y^2}$$

The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

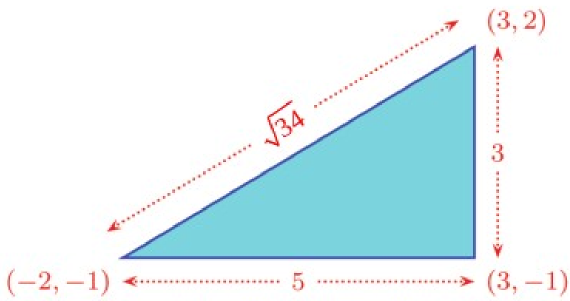
If the line joining two points is not parallel to either axis, then we can draw a rectangle with these points as opposite vertices and sides parallel to the axes





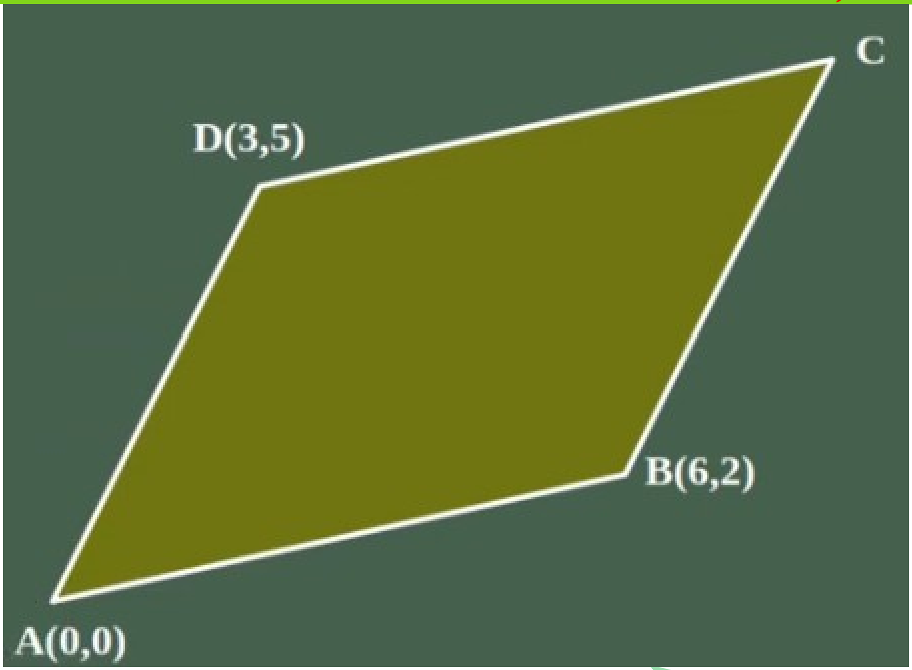
Also we know how we can find the coordinates of the other two vertices without drawing the axes.

It was using such a rectangle that we computed the distance between two points like these, in terms of their co-ordinates. We didn't use the full rectangle, but only a right triangle forming half of it.

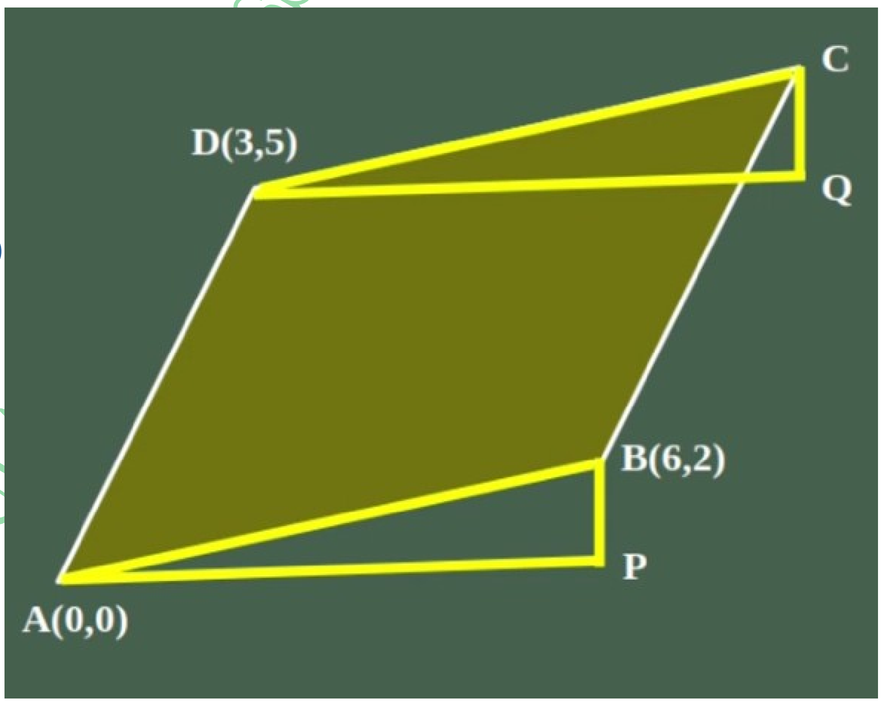


Activity

Find the fourth vertex of the parallelogram with, the origin and two other points as vertices.



Answer



In the figure ABCD is a parallelogram.

AP and DQ are parallel to x-axis. PB and QC are parallel to the y axis.

AB = CD (opposite sides of parallelogram are equal)

∟P = ∟Q = 90°

∟BAP = ∟CDQ (corresponding angles)

Therefore,

∟ABP = ∟DCQ (sum of angles of a triangle is 180°)

Therefore, ΔABP and ΔDCQ are equal triangles.

In equal triangles, sides opposite to equal angles are equal.

Therefore, AP = DQ, PB = CQ

co-ordinates of A= (0,0)

co-ordinates of B= (6,2)

The y co-ordinate of any point on the x-axis is 0. The x co-ordinate of any point on a line parallel to y-axis are same.

Therefore, co-ordinates of P = (6,0)

AP = 6 – 0 = 6

PB = 2 – 0 = 2

Therefore,

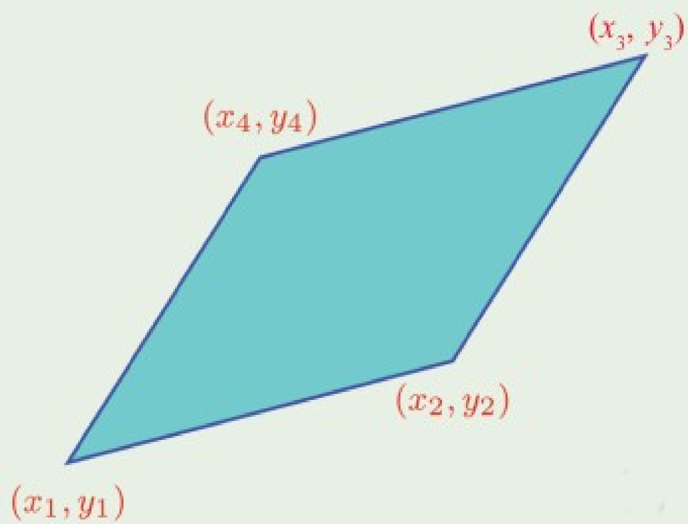
co-ordinates of Q = (3+6,5) = (9,5)

co-ordinates of C = (9,5+2) = (9,7)

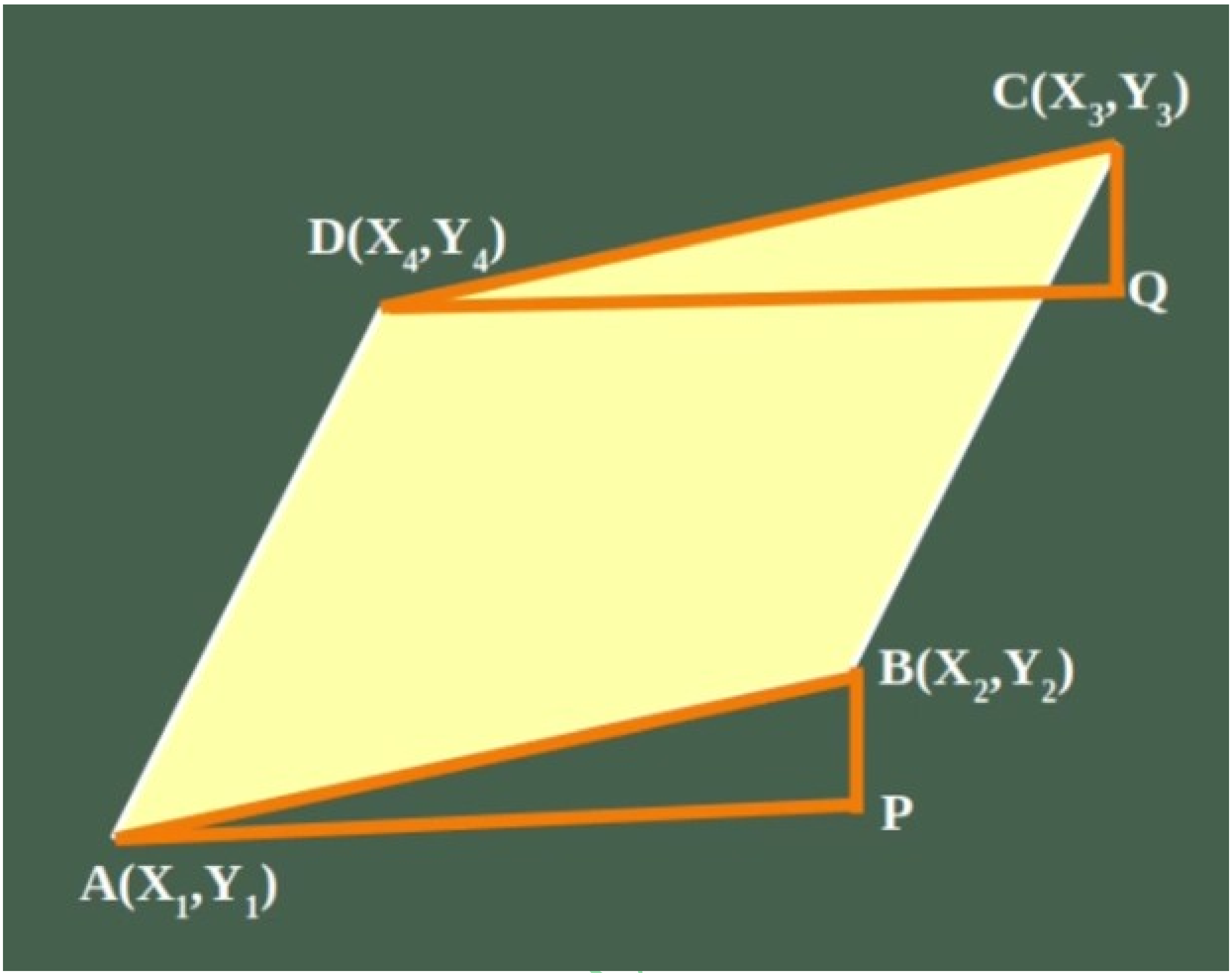
Activity

The figure shows a parallelogram with the coordinates of its vertices:

Prove that $x_1 + x_3 = x_2 + x_4$ and $y_1 + y_3 = y_2 + y_4$



Answer



In the figure ABCD is a parallelogram.

AP and DQ are parallel to x-axis. PB and QC are parallel to the y axis.

Therefore,

co-ordinates of P = (x_2, y_1)

co-ordinates of Q = (x_3, y_4)

AP = $x_2 - x_1$

DQ = $x_3 - x_4$

AP = DQ

Therefore, $x_2 - x_1 = x_3 - x_4$

$x_2 + x_4 = x_1 + x_3$

That is, $x_1 + x_3 = x_2 + x_4$

$PB = y_2 - y_1$

$QC = y_3 - y_4$

$PB = QC$

Therefore, $y_2 - y_1 = y_3 - y_4$

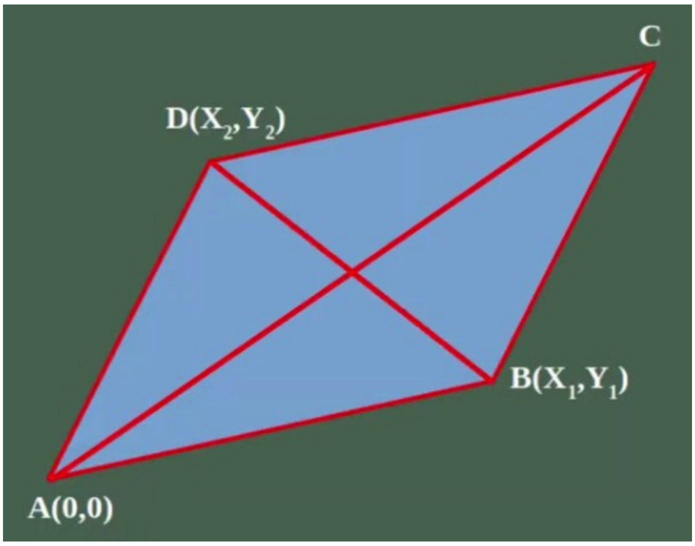
$y_2 + y_4 = y_3 + y_1$

That is, $y_1 + y_3 = y_2 + y_4$

Activity

Prove that in any parallelogram, the sum of the squares of all sides is equal to the sum of the squares of the diagonals.

Answer



Let the co-ordinates of C = (a,b)

In a parallelogram,

the sum of x co-ordinates of opposite vertices are equal.

the sum of y co-ordinates of opposite vertices are equal.

Therefore, $0 + a = x_1 + x_2$

That is, $a = x_1 + x_2$

Similarly, $0 + b = y_1 + y_2$

b = $y_1 + y_2$

Therefore, co-ordinates of C = $(x_1 + x_2 , y_1 + y_2)$

$$AB^2 = x_1^2 + y_1^2$$

$$AD^2 = x_2^2 + y_2^2$$

AD = BC

Therefore, $AD^2 = BC^2 = x_2^2 + y_2^2$

AB = CD

Therefore, $AB^2 = CD^2 = x_1^2 + y_1^2$

Therefore,

$$AB^2 + BC^2 + CD^2 + AD^2 = x_1^2 + y_1^2 + x_2^2 + y_2^2 + x_1^2 + y_1^2 + x_2^2 + y_2^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2x_1^2 + 2y_1^2 + 2x_2^2 + 2y_2^2$$

$$AC^2 = (x_1 + x_2)^2 + (y_1 + y_2)^2$$

That is, $AC^2 = x_1^2 + x_2^2 + 2x_1x_2 + y_1^2 + y_2^2 + 2y_1y_2$

$$BD^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$BD^2 = x_1^2 + x_2^2 - 2x_1x_2 + y_1^2 + y_2^2 - 2y_1y_2$$

Therefore,

$AC^2 + BD^2$ =

$$x_1^2 + x_2^2 + 2x_1x_2 + y_1^2 + y_2^2 + 2y_1y_2 + x_1^2 + x_2^2 - 2x_1x_2 + y_1^2 + y_2^2 - 2y_1y_2$$

$$= x_1^2 + x_2^2 + y_1^2 + y_2^2 + x_1^2 + x_2^2 + y_1^2 + y_2^2$$

$$= x_1^2 + y_1^2 + x_2^2 + y_2^2 + x_1^2 + y_1^2 + x_2^2 + y_2^2$$

$= \ 2x_1^2+2y_1^2+2x_2^2+2y_2^2$

$AB^2+BC^2+CD^2+AD^2=2x_1^2+2y_1^2+2x_2^2+2y_2^2$

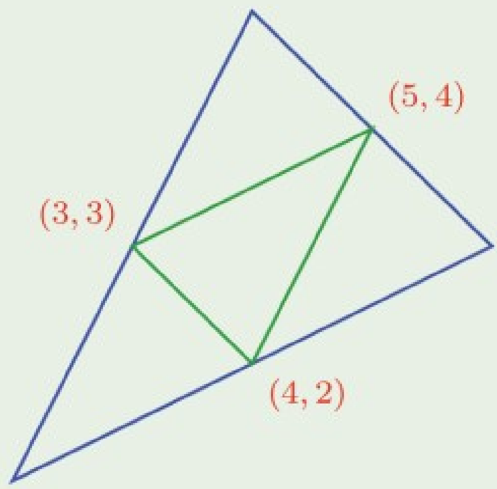
$AC^2+BD^2 \ = \ 2x_1^2+2y_1^2+2x_2^2+2y_2^2$

Therefore, $AB^2+BC^2+CD^2+AD^2 \ = \ AC^2+BD^2$

Assignment

In this picture, the mid points of the sides of the large triangle are joined to make a small triangle inside.

Calculate the coordinates of the vertices of the large triangle.

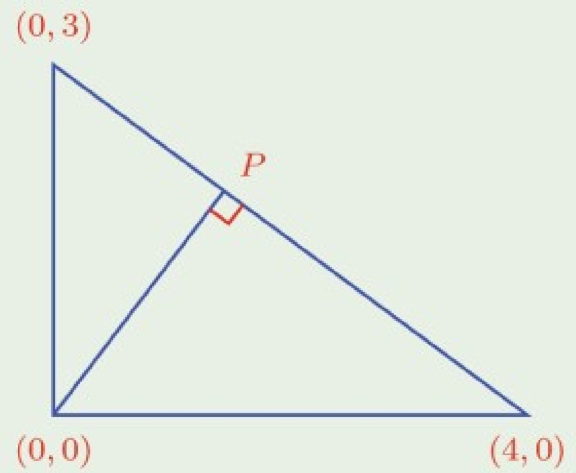


Based on the first bell class on 03-01-2021-AN

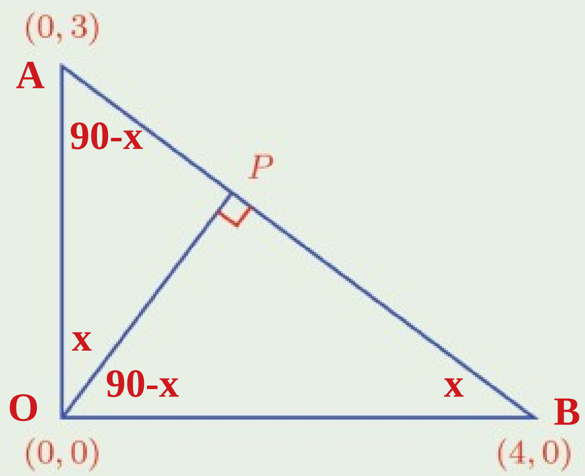
Geometry and Algebra 9

Assignment on 03-01-2020-FN

Calculate the coordinates of the point P in the picture:



Answer



In the picture,

Co-ordinates of A = (0,3)

Co-ordinates of O = (0,0)

Co-ordinates of B = (4,0)

Therefore, OA = 3

OB = 4

$$AB = \sqrt{OA^2+OB^2}$$

$$= \sqrt{3^2+4^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

Consider ΔOAP and ΔBAO.

Angles of these two triangles are equal and therefore ΔOAP and ΔBAO are similar.

Therefore, sides opposite to equal angles are proportional.

Therefore, $\frac{PA}{OA} = \frac{OA}{AB}$

That is, $\frac{PA}{3} = \frac{3}{5}$

$5 \times PA = 3 \times 3$

$PA = \frac{3 \times 3}{5} = \frac{9}{5}$

Consider ΔOAB and ΔPOB.

Angles of these two triangles are equal and therefore ΔOAB and ΔPOB are similar.

Therefore, sides opposite to equal angles are proportional.

Therefore, $\frac{OB}{PB} = \frac{AB}{OB}$

That is, $\frac{4}{PB} = \frac{5}{4}$

$5 \times PB = 4 \times 4$

$PB = \frac{4 \times 4}{5} = \frac{16}{5}$

Therefore, PA:PB = $\frac{9}{5} : \frac{16}{5}$ **= 9 : 16**

Therefore, co-ordinates of P = ($0+\frac{9}{25}(4-0), 3+\frac{9}{25}(0-3)$ **)**

$$= \left(0+\frac{9}{25}\times 4, 3+\frac{9}{25}\times -3 \right)$$

$$= \left(\frac{36}{25}, 3-\frac{27}{25} \right)$$

$$= \left(\frac{36}{25}, \frac{75-27}{25} \right)$$

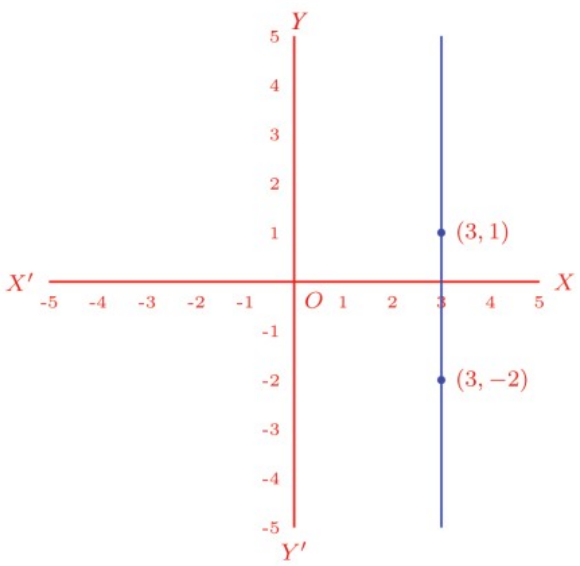
$$= \left(\frac{36}{25}, \frac{48}{25} \right)$$

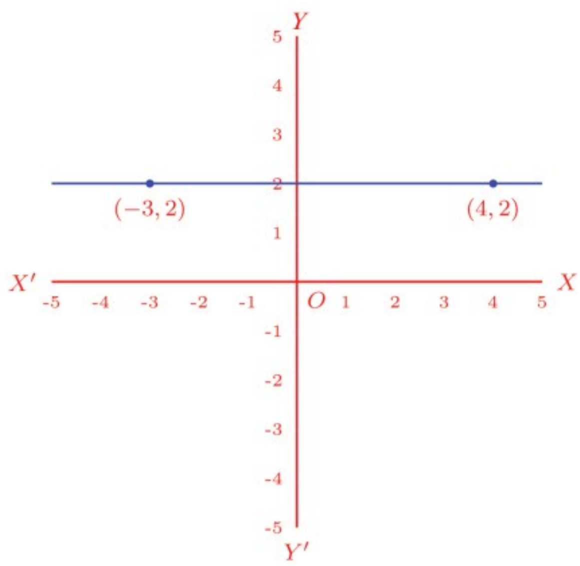
$$= \left(1\frac{11}{25}, 1\frac{23}{25} \right)$$

Straight line

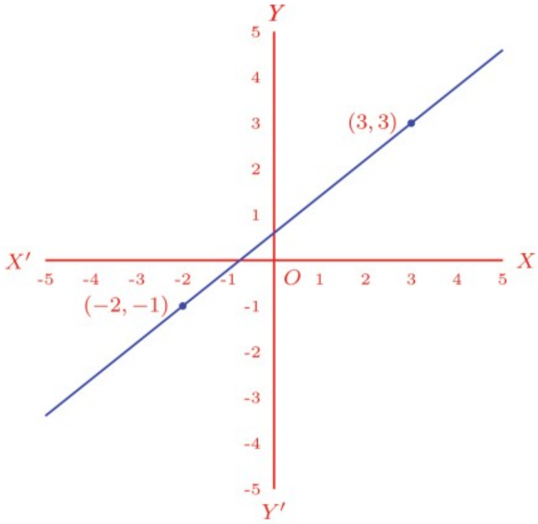
We can draw only one straight line joining any two points. we can extend this line as to either side.

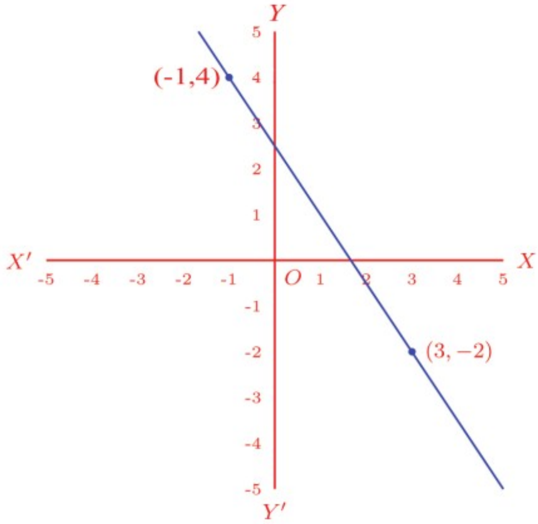
If the x coordinates of the two points are equal, then the line will be parallel to the y axis; and if the y coordinates are equal, the line will be parallel to the x axis.





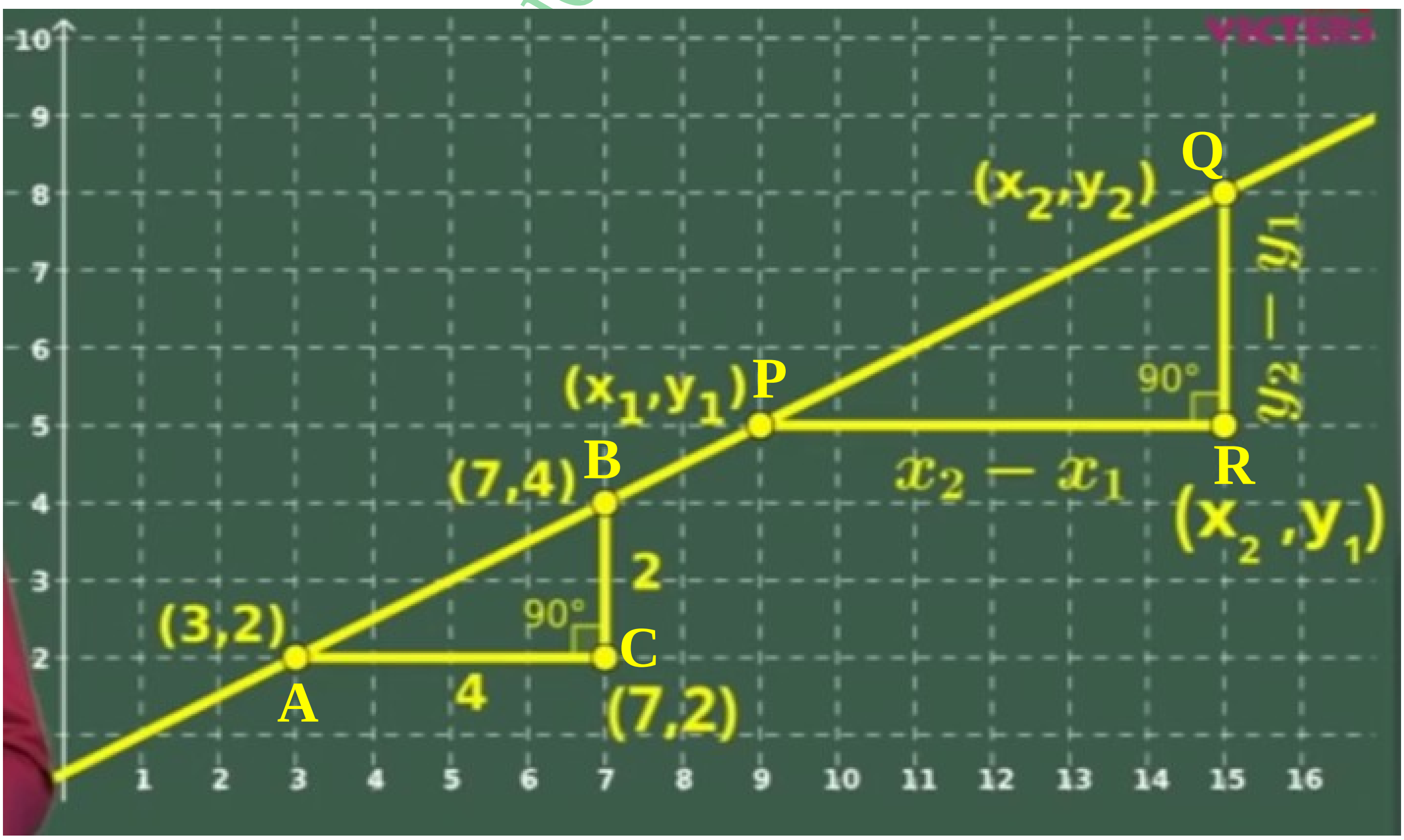
If both the x coordinates and the y coordinates are different, the line will be slanted(not parallel to either axis)





Example-1

Draw a line passing through A(3,2) and B(7,4)



Mark another two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on this line.

Draw a right triangle ACB with AB as hypotenuse and perpendicular sides(AC and BC) parallel to either axis.

Draw another right triangle PRQ with PQ as hypotenuse and perpendicular sides(PR and QR) parallel to either axis.

In ΔACB, co-ordinates of C = (7, 2)

As we move from A to B,

change in x = 7 – 3 = 4

change in y = 4 – 2 = 2

In ΔPRQ, co-ordinates of R = (x_2, y_1)

As we move from P to Q,

change in x = $x_2 - x_1$

change in y = $y_2 - y_1$

Angles of ΔACB and ΔPRQ are equal.

Therefore, these two triangles are similar.

In similar triangles, sides opposite to equal angles are proportional.

Therefore, $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{2}{4}$

That is, $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{1}{2}$

That is, $(y_2 - y_1) = \frac{1}{2}(x_2 - x_1)$

That is, the difference in y is half the difference in x.

That is, In any point on this line, change in y is half of the change in x.

Example-2

Draw a line passing through (1,3) and (3,7)

Here, As we move from (1,3) to (3,7)

change in x = 3 – 1 = 2

change in y = 7 – 3 = 4

Let (x_1, y_1) and (x_2, y_2) be another two points on this line.

change in x = $x_2 - x_1$

change in y = $y_2 - y_1$

Then we can write, $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{4}{2}$

$$\frac{(y_2 - y_1)}{(x_2 - x_1)} = 2$$

Therefore, $(y_2 - y_1) = 2(x_2 - x_1)$

That is, change in y = 2 × change in x

Example-3

Draw a line passing through (3,5) and (7,3)

Here, As we move from (3,5) to (7,3)

change in x = 7 – 3 = 4

change in y = 3 – 5 = -2

Let (x_1, y_1) and (x_2, y_2) be another two points on this line.

change in x = $x_2 - x_1$

change in y = $y_2 - y_1$

Then we can write, $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{-2}{4}$

$$\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{-1}{2}$$

$$(y_2 - y_1) = \frac{-1}{2}(x_2 - x_1)$$

That is, the difference in y = $\frac{-1}{2}$ **of the difference in x**

From these examples, we can understand that ,

> **In any line not parallel to either axis, the change in *y* coordinate is the product of the change in *x* coordinate with a fixed number**

In other words,

> **In any line not parallel to either axis, the change in *y* is proportional to the change in *x***

Note :

In a line parallel to x axis, the y coordinate does not change and so the y difference of any two points is 0.

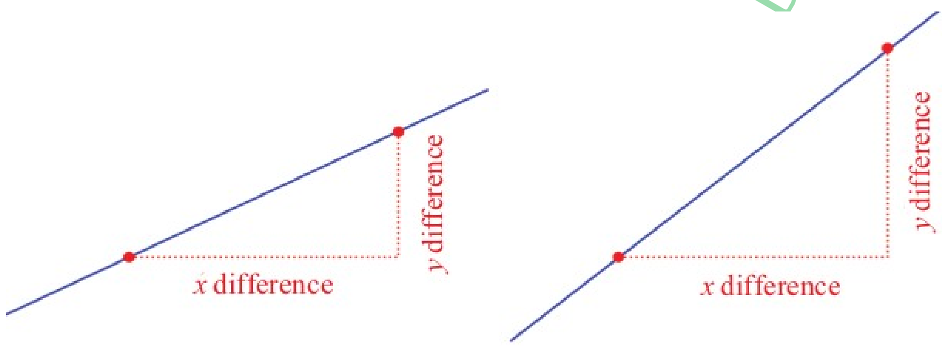
That is, $y_2 - y_1 = 0$

Here also, $y_2 - y_1$ = constant × ($x_2 - x_1$)

Here the constant = 0,

In this line the x, y change is not proportional.

Geometrically, the x difference is the horizontal shift and the y difference is the vertical shift:



So, on dividing the y difference by the x difference, we get the rate of change of the vertical shift with respect to the horizontal shift.

In other words, the constant of proportionality of the change in coordinates of a line is a measure of the slant of the line. It is called the slope of the line.

That is,

Slope of a line passing through (x_1, y_1) and (x_2, y_2) = $\frac{(y_2 - y_1)}{(x_2 - x_1)}$

In a line perpendicular to x axis or parallel to y axis, change in y is different and change in x = 0

Therefore, slope of this line = $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(y_2 - y_1)}{0}$

Division by 0 is not defined.

Therefore, the slope of a line parallel to y axis cannot be defined.

Activity

Find the co-ordinates a point on the line joining (3, 5) and (6,7).

Answer

Change in x = 6 – 3 = 3

Change in y = 7 – 5 = 2

That is, change in y = $\frac{2}{3}$ **× change in x**

Let x co-ordinate of point on this line = 4

As x increases by 1 y increases by $\frac{2}{3}$

Therefore, if x = 4 then y = 5 + $\frac{2}{3}$ **=** $5\frac{2}{3}$

Therefore, co-ordinates of a point on this line = (4, $5\frac{2}{3}$ **)**

Let x co-ordinate of another point on this line = 9

change in x = 9 – 3 = 6

change in y = 6 × $\frac{2}{3}$ **= 4**

Therefore, y co-ordinate of the point = 5 + 4 = 9

Therefore, co-ordinates of another point on this line = (9,9)

Activity

Prove that the points (1, 3), (2, 5) and (3, 7) are on the same line

Answer

Consider the points (1,3) and (2,5)

change in x = 2 - 1 = 1

change in y = 5 - 3 = 2

change in y = 2 times change in x

Consider the points (2,5) and (3,7)

change in x = 3 - 2 = 1

change in y = 7 - 5 = 2

Here also, change in y = 2 times change in x

Therefore, (1,3), (2,5), (3,7) are on the same line.

Activity

Find the coordinates of two more points on the line joining (−1, 4) and (1, 2)

Answer

Consider the points (-1,4) and (1,2)

change in x = 1- -1 = 1 + 1 = 2

change in y = 2 – 4 = -2

That is, as x increases by 2, then y decreases by 2

Therefore, a point on this line = (1+2, 2-2) = (3, 0)

One more point = (3+2, 0-2) = (5,-2)

Activity

$x_1, x_2, x_3, \ldots$ and $y_1, y_2, y_3, \ldots$ are arithmetic sequences. Prove that all the points with coordinates in the sequence $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots$ of number pairs, are on the same line

Answer

If $x_1, x_2, x_3, \ldots$ are in arithmetic sequence, then $x_2 = \frac{(x_1 + x_3)}{2}$

If $y_1, y_2, y_3, \ldots$ are in arithmetic sequence, then $y_2 = \frac{(y_1 + y_3)}{2}$

If we consider the points $(x_1, y_1), (x_2, y_2)$ then

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{\left(\frac{y_1 + y_3}{2} - y_1\right)}{\left(\frac{x_1 + x_3}{2} - x_1\right)}$$

$$= \frac{(y_1 + y_3 - 2y_1)}{(x_1 + x_3 - 2x_1)} = \frac{(y_3 - y_1)}{(x_3 - x_1)}$$

That is, $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(y_3 - y_1)}{(x_3 - x_1)}$

Therefore, $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots$ are on the same line

Activity

Find the co-ordinates of the intersecting point of the lines joining (0,2), (6,4) and (-2,6), (3,1)

Answer

Let the point of intersection is (x,y)

In the line joining (0,2) and (6,4)

change in y = 4 – 2 = 2

change in x = 6 – 0 = 6

Therefore, slope = $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{2}{6} = \frac{1}{3}$

If $((x_1, y_1)$ is a point on the line, $(y_2 - y_1) = \frac{1}{3}(x_2 - x_1)$

Therefore, $(y - 2) = \frac{1}{3}(x - 0)$

$$(y - 2) = \frac{1}{3}x$$

$$3(y - 2) = x$$

$$3y - 6 = x$$

$$x - 3y = -6$$ 1

In the line joining (-2,6) and (3,1)

change in y = 1 – 6 = -5

change in y = 3 - - 2 = 5

slope = $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{-5}{5}$ **= -1**

That is, $(y_2 - y_1) = -1(x_2 - x_1)$

Therefore, $(y-6) = -1(x - -2)$

$y - 6 = -1(x+2)$

$y - 6 = -x - 2$

$x + y = -2 + 6$

$x + y = 4$ .... (2)

$x - 3y = -6$ .... (1)

(2) - (1) → **x – x + y - -3y = 4 - -6**

y + 3y = 4 + 6

4y = 10

y = $\frac{10}{4} = \frac{5}{2} = 2\frac{1}{2}$

x + y = 4

x + $\frac{5}{2}$ **= 4**

x = 4 - $\frac{5}{2} = \frac{8-5}{2}$

$$= \frac{3}{2} = 1\frac{1}{2}$$

Therefore,

co-ordinates of the intersecting point = ($1\frac{1}{2}$, $2\frac{1}{2}$)

Assignment-1

(5, 3), (8, 9) are two points on a line.

(a) Find the slope of the line.

(b) Find the co ordinates of two other points on this line.

Assignment-2

Find the point of intersection of the lines joining the points (5, 0), (0, 5) and (6, 1), (2, -3).

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Assignments on 04-01-2020-AN

(5, 3), (8, 9) are two points on a line.

(a) Find the slope of the line.

(b) Find the co ordinates of two other points on this line.

Answer

(a) Two points on the line are (5,3), (8,9)

Slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{9-3}{8-5} = \frac{6}{3} = 2$

(b) y difference = 6

x difference = 3

That is, as x increases by 3, y increases by 6.

Therefore,

Co-ordinates of a point on this line = (8+3, 9+6) = (11, 15)

Co-ordinates of another point on this line = (11+3, 15+6)

= (14, 21)

Assignment-2

Find the point of intersection of the lines joining the points (5, 0), (0, 5) and (6, 1), (2, -3).

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Answer

Let the point of intersection is (x,y)

In the line joining (5,0) and (0,5)

change in y = 5 – 0 = 5

change in x = 0 – 5 = -5

Slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5}{-5}$ **= -1**

(x,y) is a point on this line

Therefore, $\frac{y-0}{x-5}$ **= -1**

Therefore, $y - 0 = -1(x-5)$

$y = -x + 5$

That is, $x + y = 5$ .... (1)

slope of the line joining (6,1) and (2, -3) = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1 - -3}{6-2} = \frac{4}{4}$ **= 1**

(x,y) is a point on this line.

Therefore, $\frac{y-1}{x-6}$ **= 1**

$$y-1=x-6$$

$$x-6=y-1$$

$$x-y=-1+6$$

$$x-y=5 \quad \ldots\ldots\ldots\ldots (2)$$

$$x+y=5 \quad \ldots\ldots\ldots\ldots (1)$$

Adding these two equations we get,

$$2x=10$$

$$x=\frac{10}{2}=5$$

Subtract second equation from the first we get,

$$2y=0$$

$$y=\frac{0}{2}=0$$

Therefore, point of intersection = (5, 0)

Equation of a line

Example-1

Find the equation of a line passing through (1,2) and (4,3).

Answer

Two points on the line are (1,2) and (4,3).

y difference = 3 – 2 = 1

x difference = 4 – 1 = 3

Let (x, y) is a point on this line.

Therefore,

$$\frac{y-2}{x-1} = \frac{1}{3}$$

$$3(y-2)=x-1$$

$$3y-6=x-1$$

$$x-1=3y-6$$

$$x-3y-1+6=0$$

$x-3y+5=0$ **which is the equation of the line.**

Note:

If (p,q) is a point on this line, then $p-3q+5=0$

To check (4,3) is a point on this line; substitute x = 4 and y = 3 in the equation $x-3y+5=0$

That is, $x-3y+5 = 4-3\times3+5=4-9+5=9-9=0$

Therefore, (4,3) is a point on this line.

To check (10,5) is a point on this line; substitute x = 10 and y = 5 in the equation $x-3y+5=0$

$x-3y+5 = 10-3\times5+5 = 10-15+5 = 15-15=0$

Therefore, (10, 5) is a point on this line.

Every point on this line satisfy this equation.

Example-2

Find the equation of a line passing through (0,0) and (1,1).

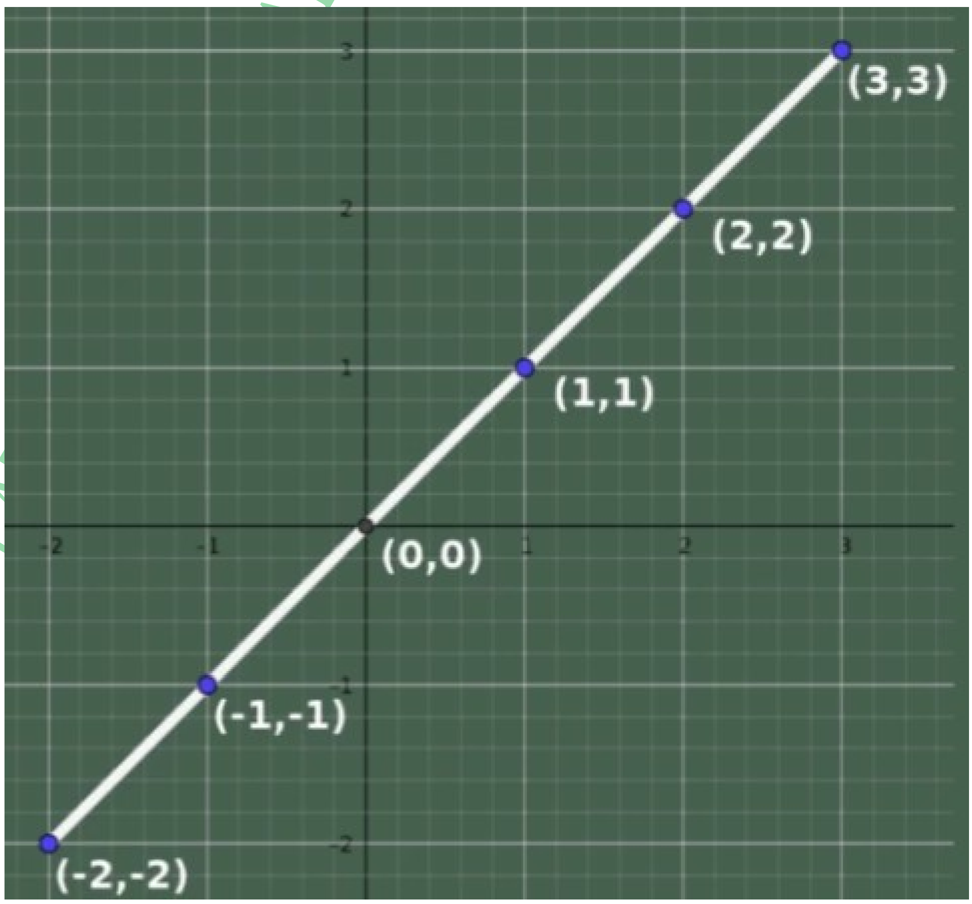
Answer

Slope = $\frac{y\,difference}{x\,difference} = \frac{1-0}{1-0} = \frac{1}{1} = 1$

Let (x, y) is a point on this line.

$$\frac{y-0}{x-0} = 1$$

$$\frac{y}{x} = 1$$



Therefore, $x = y$

This means that, in every point on this line both x and y coordinates are equal. This can also be written as $x - y = 0$

Activity

Find the equation of the line joining (1, 2) and (2, 4). For points on this line with consecutive natural numbers 3, 4, 5, ... as x coordinates, what is the sequence of y coordinates?

Answer

Two point on this line are (1,2) and (2,4)

y difference = 4 – 2 = 2

x difference = 2 – 1 = 1

Slope = $\frac{y\,difference}{x\,difference} = \frac{2}{1} = 2$

Let (x, y) is a point on this line.

Therefore, $\frac{y-2}{x-1} = 2$

$$(y-2)=2(x-1)$$

$$y-2=2x-2$$

$y=2x$ **which means that, y co-ordinate of any point on this line is 2 times the x co-ordinate.**

When x = 3 y = 6

When x = 4 y = 8

When x = 5 y = 10

That is, when x co-ordinates are 3, 5, 7, ... ;

the y coordinates are 6, 8, 10, ...

Activity

Find the equation of the line joining (–1, 3) and (2, 5). Prove that if the point (x, y) is on this line, so is the point $(x + 3, y + 2)$.

Answer

Two points on this line are (-1, 3) and (2, 5).

y difference = 5 – 3 = 2

x difference = 2 – -1 = 2 + 1 = 3

Slope = $\frac{y\,difference}{x\,difference} = \frac{2}{3}$

Let (x, y) is a point on this line.

Therefore, $\frac{y-5}{x-2} = \frac{2}{3}$

$3(y-5)=2(x-2)$

$3y-15=2x-4$

$2x-4=3y-15$

$2x-3y-4+15=0$

$2x-3y+11=0$ **which is the equation of the line.**

To check (x+3, y+2) is a point on this line, substitute x = x+3 and y = y+2 in the equation $2x-3y+11=0$

$2(x+3)-3(y+2)+11=0$

$2x+6-3y-6+11=0$

$2x-3y+11=0$

Therefore, (x+3, y+2) is a point on this line.

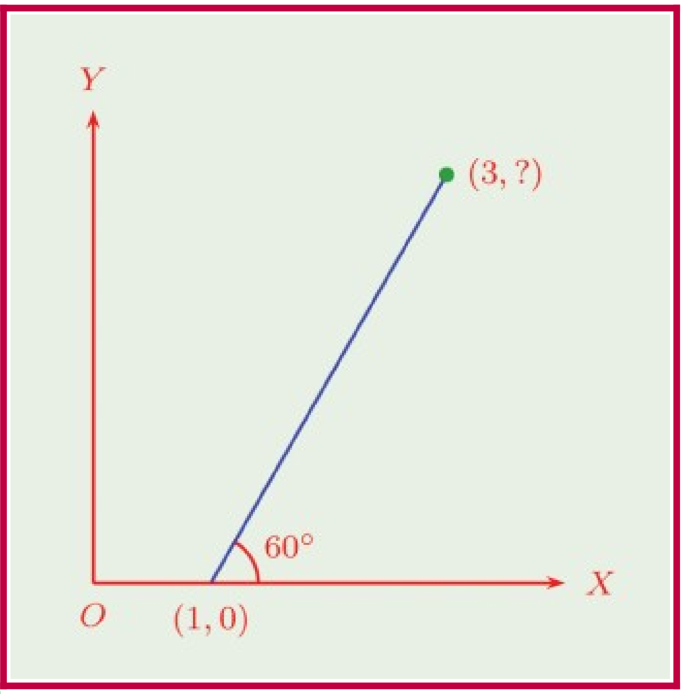
Activity

In the picture below, the x coordinate of a point on the slanted (blue) line is 3:

i) What is its y coordinate?

ii) What is the slope of the line?

iii) Write the equation of the line.



Answer

In the picture,

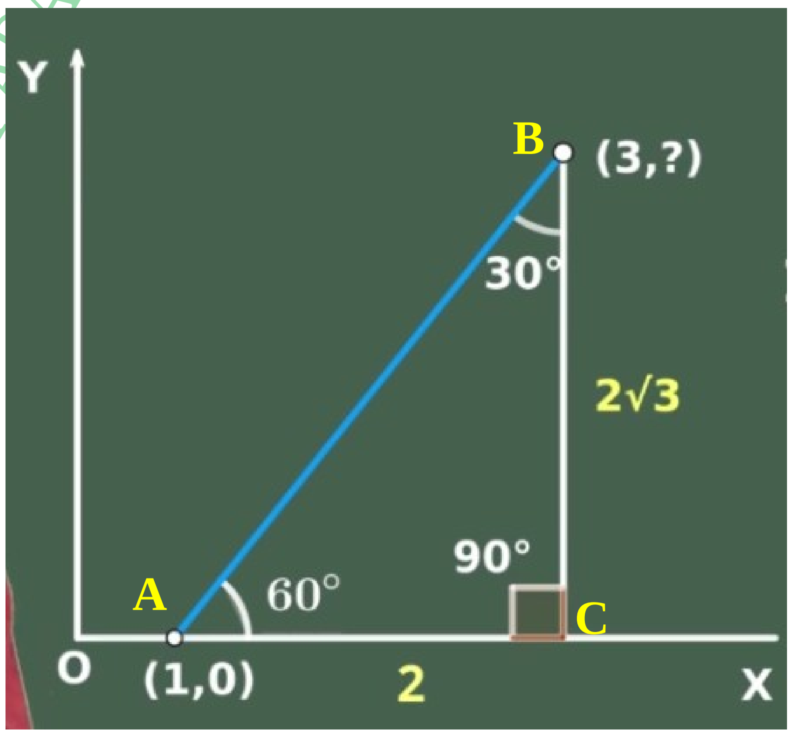
Co-ordinates of A = (1,0)

x co-ordinate of B = 3

Therefore,

Co-ordinates of C = (3,0)

Draw BC perpendicular to the x axis.



In ΔBAC,

AC = 3 – 1 = 2

$\angle$BAC = 60°

$\angle$ABC = 30°

That is, angles of ΔBAC are 30°, 60°, 90°. Its sides are in the ratio $1:\sqrt{3}:2$

Therefore, BC = $2\sqrt{3}$

Therefore,

(i) co-ordinates of B = ($3, 2\sqrt{3}$ **)**

(ii) slope of the line = $\frac{y\,difference}{x\,difference} = \frac{2\sqrt{3}-0}{3-1} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

(iii) Let (x,y) be a point on the line AB.

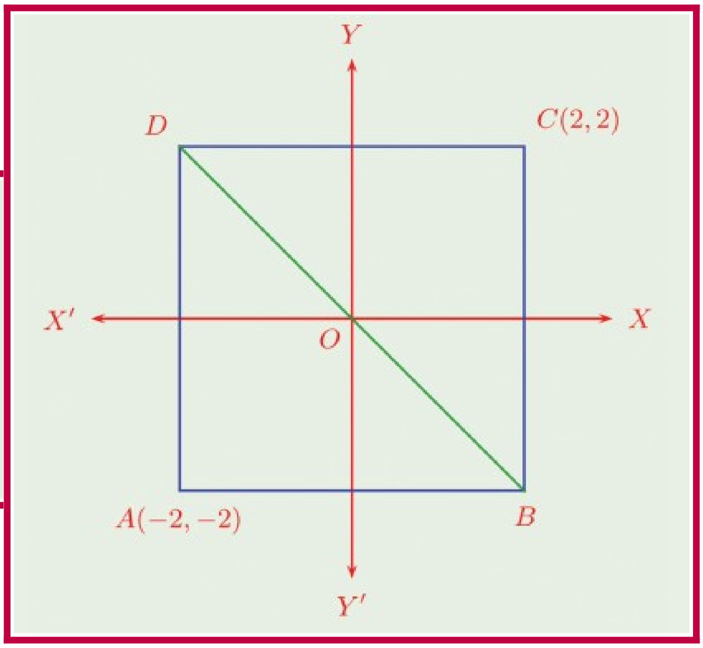
Therefore, y – 0 = $\sqrt{3}$ **(x – 1)**

That is, y = $\sqrt{3}$ **x –** $\sqrt{3}$

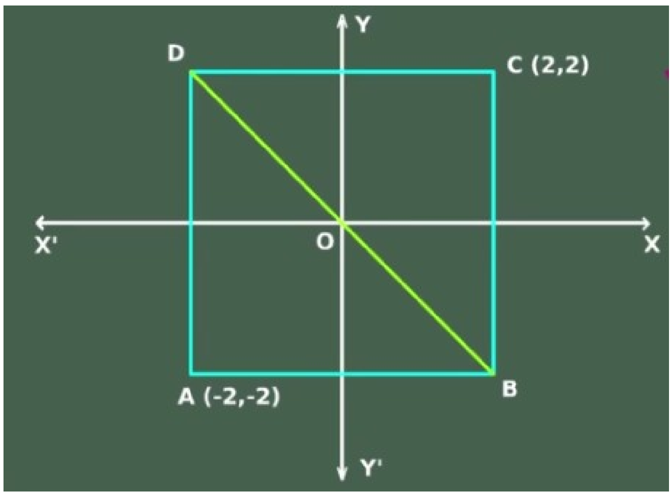
That is, $\sqrt{3}$ **x – y -** $\sqrt{3}$ **= 0 is the equation of AB**

Activity

In the picture here, *ABCD* is a square: Prove that for any point on the diagonal *BD*, the sum of the *x* and *y* coordinates is zero.



Answer



In the figure,

co-ordinate of A = (-2, -2)

co-ordinate of C = (2, 2)

Therefore,

co-ordinate of B = (2, -2)

co-ordinate of D = (-2, 2)

co-ordinate of O = (0, 0)

In the line BD,

y difference = -2 – 2 = - 4

x difference = 2 - – 2 = 2 + 2 = 4

Slope = $\frac{y\,difference}{x\,difference} = \frac{-4}{4} = -1$

Let (x,y) is a point on this line.

Therefore, $\frac{y-0}{x-0} = -1$

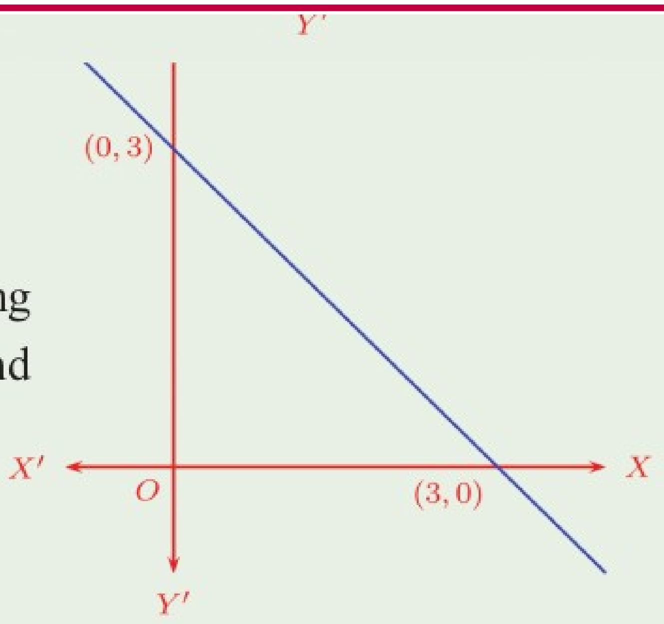
$y-0=-1(x-0)$

$y=-x$

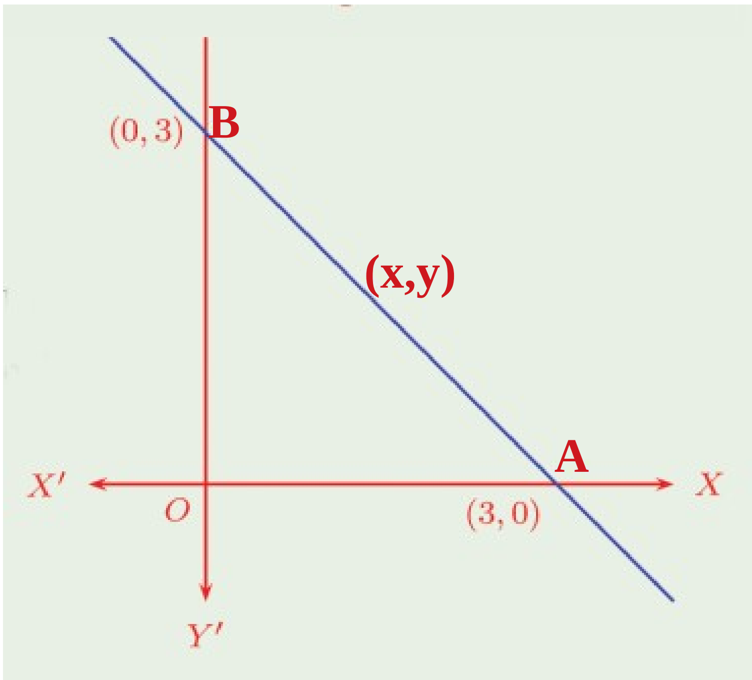
$x+y=0$ **which means that in every point on this line, the sum of x and y co-ordinates is 0.**

Activity

Prove that for any point on the line intersecting the axes in the picture, the sum of the *x* and *y* coordinates is 3.



Answer



In the picture,

co-ordinates of A = (3,0)

co-ordinates of B = (0,3)

Slope of AB = $\frac{y\,difference}{x\,difference}$

$= \frac{3-0}{0-3} = \frac{3}{-3} = -1$

Let (x,y) is a point on this line.

Therefore, $\frac{y-0}{x-3} = -1$

$y-0=-1(x-3)$

$y=-x+3$

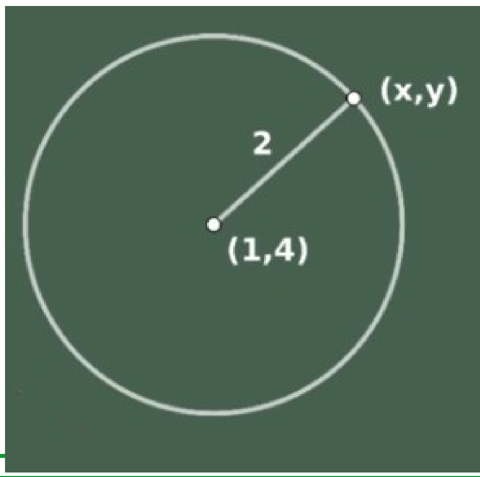
That is, $x+y=3$ which means that, in every point on this line, the sum of x and y co-ordinates is 3.

Equation of a circle

Activity

Find the equation of a circle with centre (1,4) and radius 2.

Answer



Let (x,y) is a point on the circle.

Distance between (1,4) and (x,y) = 2

That is, $\sqrt{(x-1)^2+(y-4)^2}=2$

Therefore, $(x-1)^2+(y-4)^2=2^2$

That is, $x^2-2x+1+y^2-8y+16=4$

$x^2+y^2-2x-8y+1+16=4$

$x^2+y^2-2x-8y+17=4$

$x^2+y^2-2x-8y+17-4=0$

$x^2+y^2-2x-8y+13=0$ **which means that, for every point on the circle, this equation will satisfy.**

Activity

Find the equation of a circle with centre (0,0) and radius 1.

Answer

Let (x,y) is a point on the circle.

Distance between (0,0) and (x,y) = 1

That is, $\sqrt{(x-0)^2+(y-0)^2}=1$

$(x-0)^2+(y-0)^2=1$

$x^2+y^2=1$

That is, Equation of a circle with origin as centre and radius 1 is $x^2+y^2=1$

Similarly,

Equation of a circle with origin as centre and radius 2 is

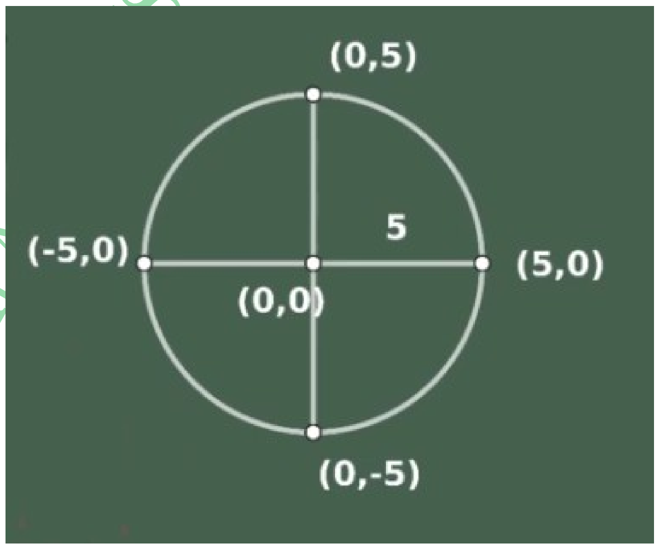
$x^2+y^2=2^2$

That is, $x^2+y^2=4$

Activity

Find the equation of the circle with centre at the orgin and radius 5. Write the coordinates of eight points on this circle.

Answer



Equation of a circle with origin as centre and radius 5 is $x^2+y^2=5^2$

That is, $x^2+y^2=25$

Radius of the circle = 5

Therefore, the co-ordinates of 4 points which cut the axes are (5,0), (0,5), (-5,0), (0,-5).

$x^2+y^2=25$ **which means that the sum of squares of two numbers is 25.**

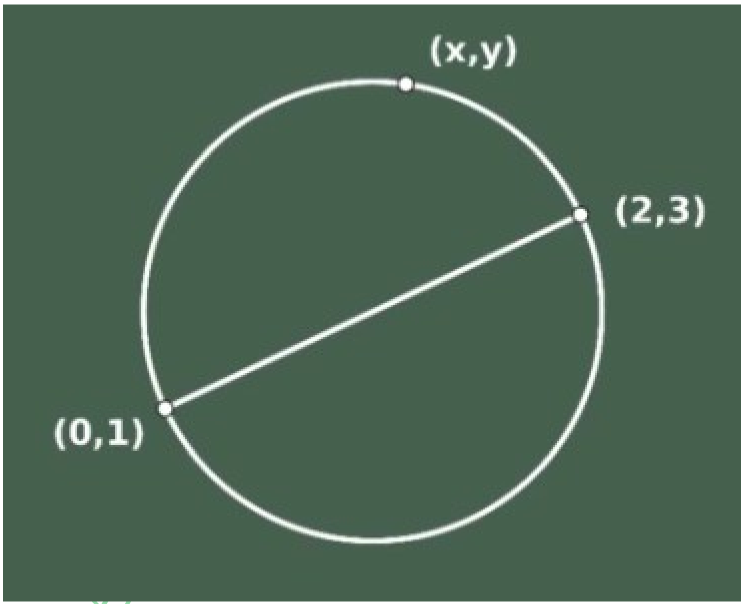
We know that $3^2+4^2=25$

Therefore, x = 3 and y = 4

Now the co-ordinates of another 4 points on the circle are (3,4), (3,-4), (-3,4) and (-3,-4)

Activity

Prove that if (x, y) be a point on the circle with the line joining (0, 1) and (2, 3) as diameter, then $x^2 + y^2 - 2x - 4y + 3 = 0$. Find the coordinates of the points where this circle cuts the y axis.



Answer

In the figure, AB is a diameter of the circle.

Co-ordinates of A = (0,1)

Co-ordinates of B = (2,3)

Centre of the circle = midpoint of the diameter = ($\frac{(x_1+x_2)}{2}$, $\frac{(y_1+y_2)}{2}$)

= ($\frac{(0+2)}{2}$, $\frac{(1+3)}{2}$)

= ($\frac{2}{2}$, $\frac{4}{2}$) = (1,2)

radius = $\sqrt{(1-0)^2+(2-1)^2}$ = $\sqrt{1^2+1^2}$ = $\sqrt{1+1}$ = $\sqrt{2}$

Let (x,y) is a point on the circle.

The equation of the circle is

$(x-1)^2+(y-2)^2=(\sqrt{2})^2$

$x^2-2x+1+y^2-4y+4=2$

$x^2+y^2-2x-4y+5-2=0$

$x^2+y^2-2x-4y+3=0$

If the circle cuts the y axis, the x co-ordinate of that points are zero.

Therefore, substitute x = 0 in the equation

$x^2+y^2-2x-4y+3=0$

That is, $0^2+y^2-2\times0-4y+3=0$

That is, $y^2-4y+3=0$

That is, $(y-1)(y-3)=0$

That is, $(y-1)=0$ **or** $(y-3)=0$

That is, $y=1$ **or** $y=3$

Therefore, co-ordinates points which cuts the y axis are (0,1) and (0,3).

Assignment-1

Find the equation of the line joining (1,4) and (6,6).

Assignment-2

Find the equation of the circle with the line joining (2,0) and (0,4) as diameter.

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