CHAPTER-1
ELECTRIC CHARGES AND FIELDS

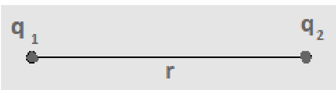
(Prepared by AYYAPPAN C, HSST, GMRHSS KASARAGOD)

Electric Charge

- **Electric charge** is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.
- **The two types of charges are positive and negative (Named by Benjamin Franklin)**
- **Like charges** repels and unlike charges attracts.
- When amber rubbed with wool or silk cloth attracts light objects – discovered by Thales.
- **Electroscope** – device for charge detection
- It is a **scalar quantity** .
- SI unit of electric charge- **coulomb (C)**
- Charge of a proton is positive **(1.602192×10^{-19} C)**
- Charge of an electron is negative **($-1.602192 \times 10^{-19}$ C)**
- Matter with **equal number of electrons and protons** are **electrically neutral.**
- Matter with excess number of electrons – negatively charged
- Matter with excess protons – positively charged.

Coulomb's law

- The force of attraction or repulsion between two stationary electric charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.



- Force between two stationary charges is

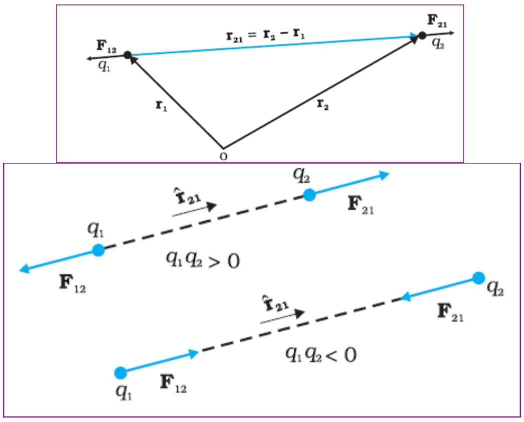
$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2}$$

- Where ε_0 -permittivity of free space, ε_r - relative permittivity.
- Relative permittivity is given by , $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$
- ε - Permittivity of the medium.
- Also $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$
- Thus $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

Definition of coulomb

- *When* $q_1 = q_2 = 1$ C, $r = 1$ m , $F = 9 \times 10^9$ N
- 1 C is the charge that when placed at a distance of 1 m from another charge of the same magnitude *in vacuum* experiences an electrical force of repulsion of magnitude 9×10^9 N.

Coulomb's law in vector form



- Force on q_1 due to q_2 is,

$$\vec{F}_{12} = \frac{1}{4\Pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

- Force on q_2 due to q_1 is,

$$\vec{F}_{21} = \frac{1}{4\Pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

- Thus $F_{12} = -F_{21}$, Coulomb's law agrees with Newton's third law.

Electric field

- Region around a charge where its effect can be felt.
- Intensity of electric field at a point is the force per unit charge.

$$E = \frac{F}{q}$$

$$F = qE$$

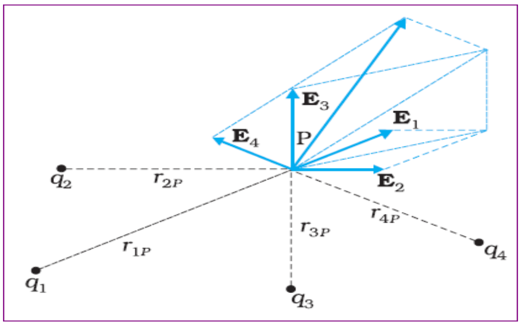
- Unit of electric field is N/C or V/m.
- It is a vector quantity.

Electric field due to a point charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Electric field due to a system of charges

- Total electric field at a point due to a system of charges is the vector sum of the field due to individual charges.



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{\mathbf{r}}_{iP}$$

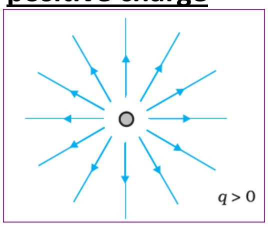
Electric field lines

- Pictorial representation of electric field.
- Electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.

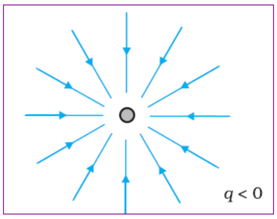
Properties of field lines

- Start from positive charge, end at negative charge. Do not form closed loops.
- Field lines are continuous in a charge free region.
- Two field lines never intersect.(Reason: two directions for electric field is not possible at a point)
- Field lines are parallel in uniform electric field.
- Tangent at any point gives direction of electric field.
- Number of field lines gives intensity of electric field.

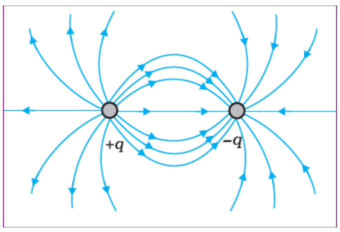
positive charge



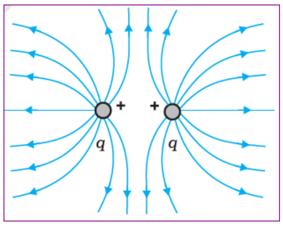
negative charge



Positive and negative charge (dipole)

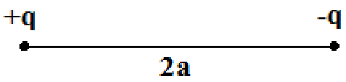


Two positive charges



Electric Dipole

- Two equal and opposite charges separated by a small distance.



- Total charge and force on a dipole is **zero.**

Dipole moment

- Product of charge and dipole length.

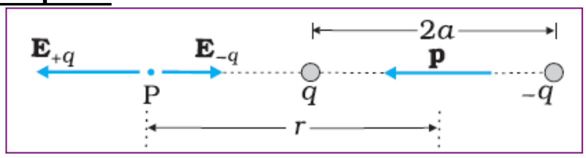
$$p = q \times 2a$$

q- charge, 2a- dipole length

- Direction is from negative to positive charge.
- SI unit- coulomb metre (C m)

Electric field due to a dipole

Axial point



- The field at the point P due to positive charge is

$$\mathbf{E}_{+q} = \frac{q}{4\pi\varepsilon_0(r-a)^2}\hat{\mathbf{p}}$$

- The field due to negative charge is

$$\mathbf{E}_{-q} = -\frac{q}{4\pi\varepsilon_0(r+a)^2}\hat{\mathbf{p}}$$

- Thus the total electric field at P is

$$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{\mathbf{p}}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r+a)^2\times(r-a)^2}\right]\hat{p}$$

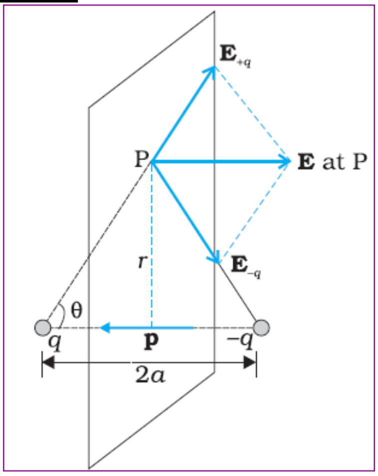
- Simplifying

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{\left(r^2-a^2\right)^2}\right]\hat{p}$$

- For r >> a, we get $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{4qa}{r^3}\right]\hat{p}$
- Using p= q x2a

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2p}{r^3}\right]\hat{p}$$

Equatorial point



- The magnitudes of the electric fields due to the two charges $+q$ and $-q$ are equal and given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2} \qquad E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2}$$

- The components normal to the dipole axis cancel away.
- The components along the dipole axis add up.
- Thus total electric field is

$$\mathbf{E} = -(E_{+q} + E_{-q})\cos\theta\ \hat{\mathbf{p}}$$

- Substituting $\cos\theta = \frac{a}{(r^2+a^2)^{\frac{1}{2}}}$ and

  simplifying we get

$$\vec{E} = \frac{-q\times 2a}{4\pi\varepsilon_0 (r^2+a^2)^{\frac{3}{2}}}\hat{p}$$

- For r >> a, we get $\vec{E} = \frac{-q\times 2a}{4\pi\varepsilon_0 r^3}\hat{p}$
- Using p= q x2a

$$\vec{E} = \frac{-p}{4\pi\varepsilon_0 r^3}\hat{p}$$

Relation connecting axial field and equatorial field of dipole

- We have axial field

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2p}{r^3}\right]\hat{p}$$

- Equatorial field

$$\vec{E} = \frac{-p}{4\pi\varepsilon_0 r^3}\hat{p}$$

- Thus

$$\mathbf{E}_{axial} = 2\times\mathbf{E}_{equatorial}$$

Physical significance of electric dipole

Non Polar molecules

- The molecules in which positive centre of charge and negative centre of charge lie at the same place.
- Dipole moment is zero for a non polar molecule in the absence of an external field.
- They develop a dipole moment when an electric field is applied.
- Eg:CO_2, CH_4, etc.

Polar molecules

- The molecules in which the centres of negative charges and of positive charges do not coincide.
- Eg: water

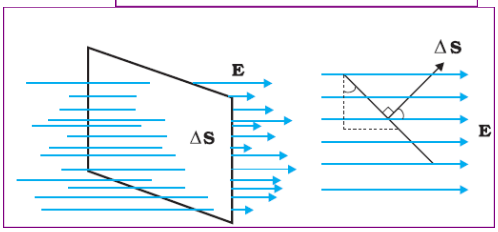
Electric flux

- Number of field lines passing normal through a surface.

$$\phi = EA\cos\theta$$

- Or

$$\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{S} = E\,\Delta S\ \cos\theta$$



- Unit – **Nm^2/ C**
- **It is a scalar quantity**

Charge density

Linear charge density (λ)

- It is the charge per unit length.

$$\lambda = \frac{Q}{l}$$

- SI unit is C/m.

Surface charge density (σ)

- It is the charge per unit area.

$$\sigma = \frac{Q}{A}$$

- SI unit is C/m^2.

Volume charge density (ρ)

- It is the charge per unit volume.

$$\rho = \frac{Q}{V}$$

- SI unit is C/m^3.

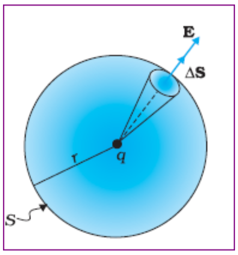
Gauss's Theorem

- Total electric flux over a closed surface is

$$\phi = \frac{q}{\varepsilon_0}$$

- Where q - total charge enclosed
- The closed surface – Gaussian surface.

Proof



- The flux through area element ΔS is

$$\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{S} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}\cdot\Delta\mathbf{S} \qquad \Delta\phi = \frac{q}{4\pi\varepsilon_0 r^2}\Delta S$$

- The total flux through the sphere is

$$\phi = \sum_{all\ \Delta S} \frac{q}{4\pi\varepsilon_0 r^2}\Delta S$$

$$\phi = \frac{q}{4\pi\varepsilon_o r^2}\sum_{all\ \Delta S}\Delta S = \frac{q}{4\pi\varepsilon_0 r^2}S$$

- Where the total surface area $S = 4\pi r^2$.
- Thus

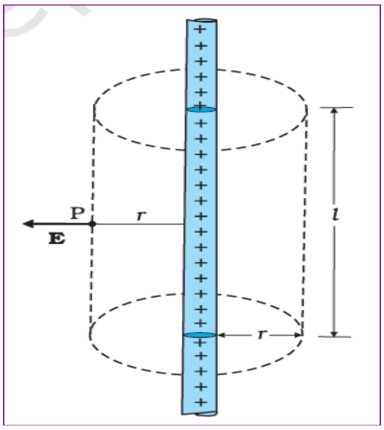
$$\phi = \frac{q}{4\pi\varepsilon_0 r^2}\times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

Features of Gauss's law

- Gauss's law is true for any closed surface irrespective of the size and shape.
- The charge includes sum of all charges enclosed by the surface.
- Gauss's law is useful to calculate electric field when the system has some symmetry.
- Gauss's law is based on the inverse square dependence on distance contained in the Coulomb's law.

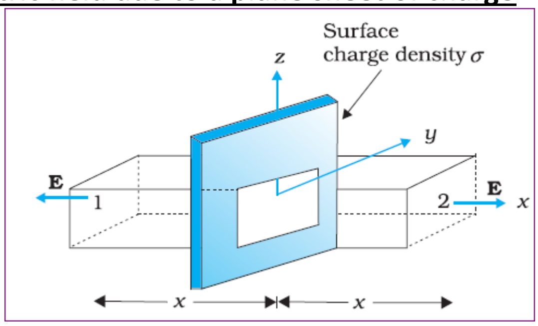
Applications of Gauss's law

Electric field due to a straight charged wire



- Total flux through the Gaussian surface is
$$\phi = E\times 2\pi rl$$
- Total charge enclosed is
$q = \lambda\times l$, λ- charge per unit length
- Using Gauss's law
$$E\times 2\pi rl = \frac{\lambda l}{\varepsilon_0}$$
- Thus $$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$
- In vector form $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$
- Where $\hat{n}$ - radial unit vector

Electric field due to a plane sheet of charge

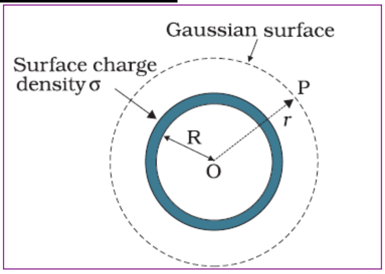


- Total flux enclosed by the Gaussian surface is $\phi = E\times(2A)$, A- area of cross section.
- Total charge enclosed is $q = \sigma A$,
σ – surface charge density.

- Using Gauss's law $E \times (2A) = \frac{\sigma A}{\varepsilon_0}$
- Thus $E = \frac{\sigma}{2\varepsilon_0}$
- In vector form $\vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{n}$
-

Electric field due to a charged spherical shell

Points outside the shell

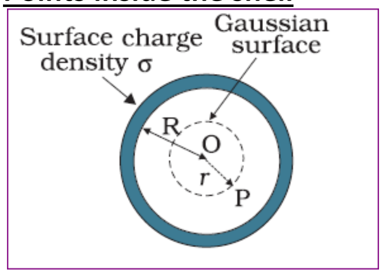


- Total flux enclosed by the Gaussian surface is $\phi = E \times (4\pi r^2)$, r- radius of Gaussian surface.
- Total charge enclosed is $q = \sigma \times (4\pi R^2)$, R –radius of shell
- Using Gauss's law $E \times (4\pi r^2) = \frac{\sigma \times 4\pi R^2}{\varepsilon_0}$
- Thus $E = \frac{\sigma R^2}{\varepsilon_0 r^2}$
- Or $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$
- In vector form $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$

Points on the shell

- On the surface r=R, therefore $E = \frac{\sigma}{\varepsilon_0}$

Points inside the shell



- Total charge enclosed =0
  $E \times 4\pi r^2 = 0$
- Thus E= 0 inside the shell.

- Vanishing of electric field (E=0) inside a charged conductor is called **electrostatic shielding**

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