Chapter Two

ELECTROSTATIC POTENTIAL AND CAPACITANCE

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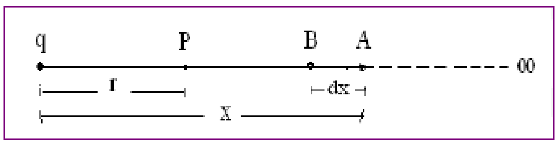
ELECTROSTATIC POTENTIAL

- The electrostatic potential (*V*) at any point is the work done in bringing a unit positive charge from infinity to that point.

  $V = \frac{W}{q}$, W – work done, q – charge.

- Also $W = qV$
- It is a scalar quantity.
- Unit is J/C or volt (V)

POTENTIAL DUE TO A POINT CHARGE



- The force acting on a unit positive charge (+1 C) at A , is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q\times 1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

- Thus the work done to move a unit positive charge from A to B through a displacement dx is

$$dW = -\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}dx$$

- The negative sign shows that the work is done against electrostatic force.
- Thus the total work done to bring unit charge from infinity to the point P is

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r}\left[-\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}dx\right]$$

$$W = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r}\left[\frac{1}{x^2}dx\right]$$

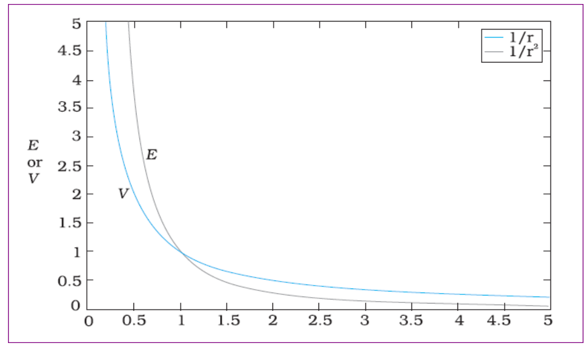
- Integrating

$$W = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{q}{4\pi\varepsilon_0 r}$$

- Therefore electrostatic potential is given by

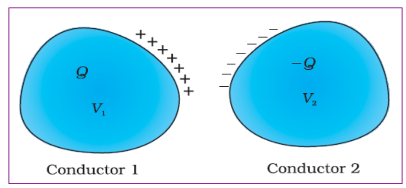
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Variation of potential *V with r*



Capacitor

- It is a charge storing device.
- A capacitor is a system of two conductors separated by an insulator.



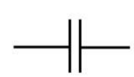
- A capacitor with large capacitance can hold large amount of charge Q *at a relatively small V.*

Capacitance

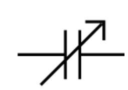
- The potential difference is proportional to the charge , Q.
- Thus $C = \frac{Q}{V}$
- The constant C is called the *capacitance of the capacitor. C is independent* of *Q or V.*
- The capacitance *C depends only on the* geometrical configuration (shape, size, separation) of the system of two conductors
- SI unit of capacitance is **farad.**
- Other units are, 1 μF = 10^{-6} F, 1 nF = 10^{-9} F, 1 pF = 10^{-12} F, etc.

Symbol of capacitor

Fixed capacitance



Variable capacitance



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Dielectric strength

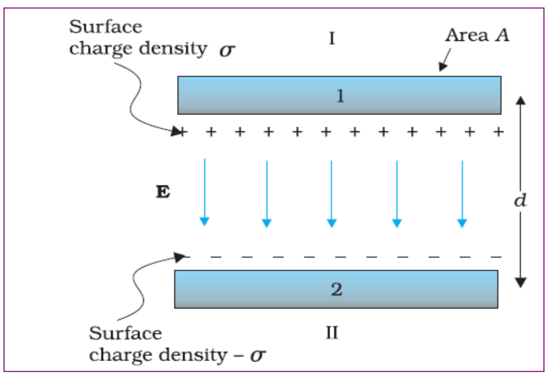
- The maximum electric field that a dielectric medium can withstand without break-down is called its dielectric strength.
- The dielectric strength of air is about $3 \times 10^6\ Vm^{-1}$.

THE PARALLEL PLATE CAPACITOR

- A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance

Capacitance of parallel plate capacitor

- Let *A be* the area of each plate and *d the separation between* them.
- The two plates have charges *Q and –Q.*
- Plate 1 has surface charge density σ = *Q/A and plate 2 has a surface* charge density –σ.



- At the region I and II, E=0

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

- At the inner region

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

- The direction of electric field is from the positive to the negative plate.
- For a uniform electric field the potential difference is

$$V = E\,d = \frac{1}{\varepsilon_0}\frac{Qd}{A}$$

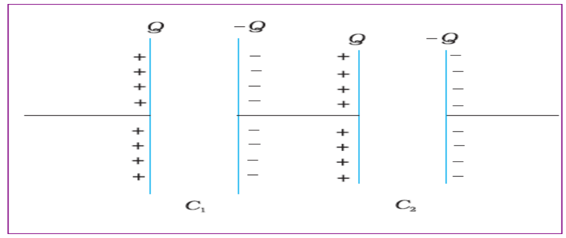
- The capacitance *C of the parallel plate capacitor is then*

$$C = \frac{Q}{V} = = \frac{\varepsilon_0 A}{d}$$

- Thus $C = \dfrac{\varepsilon_0 A}{d}$

Combination of capacitors

Capacitors in series



- In series charge is same and potential is different on each capacitors.
- The total potential drop *V* across the combination is

$$V = V_1 + V_2$$

- Considering the combination as an effective capacitor with charge *Q and potential* difference *V, we get*

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

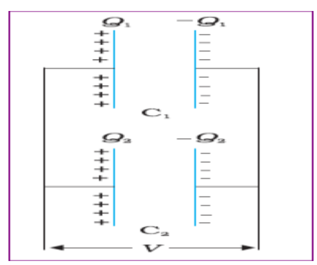
- Therefore effective capacitance is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

- For n capacitors in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots + \frac{1}{C_n}$$

Capacitors in parallel



- In parallel the charge is different, potential is same on each capacitor.
- The charge on the equivalent capacitor is

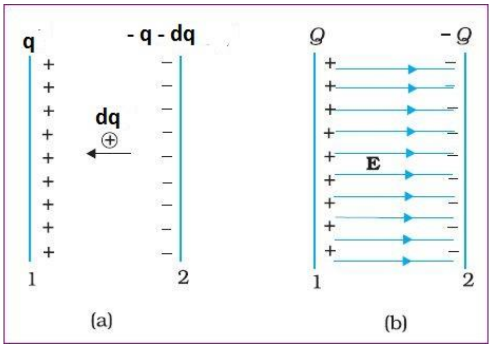
$$Q = Q_1 + Q_2$$

- Thus $CV = C_1V + C_2V$
- Therefore $C = C_1 + C_2$
- In general , for n capacitors

$$C = C_1 + C_2 + \ldots\ldots\ldots\ldots + C_n$$

Energy stored in a capacitor

- Energy stored in a capacitor is the **electric potential energy.**



- Charges are transferred from conductor 2 to conductor 1 bit by bit, so that at the end, conductor 1 gets charge *Q.*
- Work done to move a charge **dq** from conductor 2 to conductor 1, is
  $dW = Potential \times Charge$
- That is $dW = \frac{q}{C} \times dq$
- Since potential at conductor 1 is , **q/C.**
- Thus the total work done to attain a charge Q on conductor 1, is

$$W = \int_0^Q dW = \int_0^Q \frac{q}{C} \times dq$$

- On integration we get,

$$W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

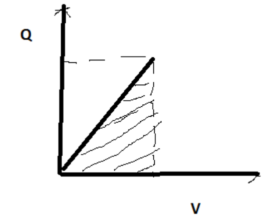
- This work is stored in the form of potential energy of the system.
- Thus energy stored in the capacitor is

$$U = \frac{Q^2}{2C}$$

- Also $U = \frac{1}{2}QV$ or $U = \frac{1}{2}CV^2$

Alternate method

- We have the Q – V graph of a capacitor,



- Energy = area under the graph
- Thus , $U = \frac{1}{2} \times Q \times V$
- Also $U = \frac{1}{2}CV^2$

Energy Density of a capacitor

- Energy density is the energy stored per unit volume.
- We have $U = \frac{Q^2}{2C}$
- But $Q = \sigma A$ and $C = \frac{\varepsilon_0 A}{d}$
- Thus we get $U = \frac{(\sigma A)^2}{2}\left(\frac{d}{\varepsilon_0 A}\right)$
- Using $E = \frac{\sigma}{\varepsilon_0}$, we get
  $U = \frac{1}{2}\varepsilon_0 E^2 \times Ad$
- Thus energy per unit volume is given by
  $$\frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$
- That is the energy density of the capacitor is
  $$u = \frac{1}{2}\varepsilon_0 E^2$$

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