CHAPTER 4
MOVING CHARGES AND MAGNETISM

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Magnetic Lorentz force

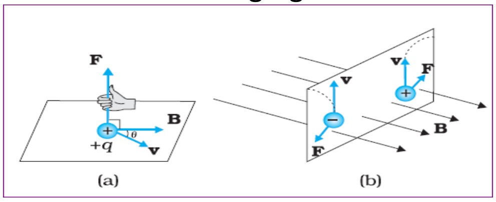
- Force on charge moving in a magnetic field.
  $F = qvB\sin\theta$, q –charge, v- velocity, B – magnetic field, θ- angle between v and B.
- Or $F = q(v \times B)$

Special Cases:

- **If the charge is at rest**, i.e. v = 0, then F = 0.
- Thus, a stationary charge in a magnetic field does not experience any force.
- **If θ = 0° or 180°** i.e. if the charge moves parallel or anti-parallel to the direction of the magnetic field, then F = 0.
- **If θ = 90°** i.e. if the charge moves perpendicular to the magnetic field, then the force is maximum.
  $$F_{\max} = qvB$$

Right Hand Thumb Rule

- The direction of magnetic Lorentz force can be found using right hand rule.



Work done by magnetic Lorentz force

- The magnetic Lorentz force is given by $F = q(v \times B)$
- Thus F, is perpendicular to v and hence perpendicular to the displacement.
- Therefore the work done
  $W = Fd\cos 90 = 0$
- Thus **work done by the magnetic force** on a moving charge is **zero.**
- The change in kinetic energy of a charged particle, when it is moving through a magnetic field is zero.
- The magnetic field can change the direction of velocity of a charged particle, but not its magnitude.

Lorentz force

- Force on charge moving in combined electric and magnetic field.
- $F = qE + q(v \times B) = q\left[E + (v \times B)\right]$

Units of magnetic field (magnetic induction or magnetic flux density)

- SI unit is **tesla (T)**
- Other unit is **gauss(G)**
- **1 gauss =10^{-4} tesla**
- **The earth's magnetic field is about 3.6×10^{-5} T**

Definition of Tesla

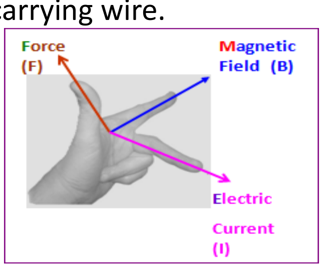
- The magnetic induction (B) in a region is said to be one tesla if the force acting on a unit charge (1C) moving perpendicular to the magnetic field (B) with a speed of 1m/s is one Newton.

Force on a current carrying wire in a magnetic field

- The total number of charge carriers in the conductor = n Al
- Where, n-number of charges per unit volume, A-area of cross section, l-length of the conductor.
- If e is the charge of each carrier , the total charge is $Q = enAl$
- The magnetic force is $F = Q(v \times B)$
- Where v – drift velocity
- Thus $F = enAl(v \times B) = nAve(l \times B)$
- Thus
  $$F = IlB\sin\theta$$
- Since $I = nAve$
- When θ=0 , F=0
- When θ=90^0, F = I/B

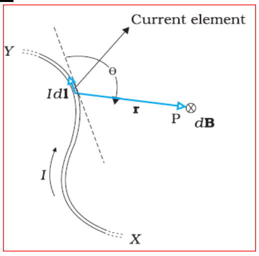
Fleming's left hand rule

- A rule to find the direction of the force n a current carrying wire.



- Fore finger – direction of magnetic field
- Middle finger –direction of current
- Thumb – direction of force.

Biot-Savart Law



- The magnetic field at a point due to the small element of a current carrying conductor is
- directly proportional to the current flowing through the conductor (I)
- The length of the element dl
- Sine of the angle between r and dl
- And inversely proportional to the square of the distance of the point from dl.
- Thus the magnetic field due to a current element is

$$\mathbf{dB} = \frac{\mu_0}{4\pi}\frac{\mathbf{Idl\sin\theta}}{\mathbf{r}^2}$$

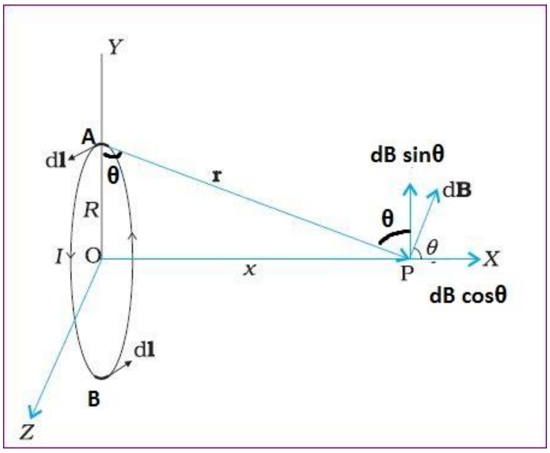
- μ_0-permeability of free space, I – current, r- distance
- or $dB = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$
- where , $\frac{\mu_0}{4\pi} = 10^{-7}Tm/A$
- The direction of magnetic field is given by right hand rule.

Comparison between Coulomb's law and Biot-Savart's law

Coulomb's law	Biot – Savart's law
$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2}$	$\mathbf{dB} = \frac{\mu_0}{4\pi}\frac{\mathbf{Idl\sin\theta}}{\mathbf{r}^2}$
Electric field is due to scalar source	Magnetic field is due to vector source
Electric field is present everywhere	Along the direction of current magnetic field is zero

Applications of Biot-Savart Law

Magnetic Field on the Axis of a Circular Current Loop



- The magnetic field at P due to the current element dl , at A is

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin 90^0}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

- The component dB sin**θ** is cancelled by the diametrically opposite component.
- Thus magnetic field at P ,due to the current element is the x- component of dB.
- Therefore $dB_x = dB\cos\theta$

$$dB_x = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\cos\theta$$

- But we have $r = \left(x^2+R^2\right)^{1/2}$ and

$$\cos\theta = \frac{R}{\left(x^2+R^2\right)^{1/2}}$$

- Therefore

$$dB_x = \frac{\mu_0}{4\pi}\frac{Idl}{(x^2+R^2)}\frac{R}{(x^2+R^2)^{1/2}}$$

$$dB_x = \frac{\mu_0}{4\pi}\frac{IRdl}{(x^2+R^2)^{3/2}}$$

- The summation of the current elements dl over the loop gives , the circumference 2πR.
- Thus the total magnetic field at P due to the circular coil is

$$B = \frac{\mu_0}{4\pi}\frac{IR(2\pi R)}{(x^2+R^2)^{3/2}} = \frac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}}$$

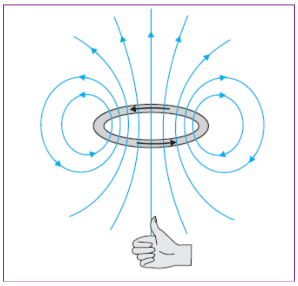
- Therefore

$$B = \frac{\mu_0 IR^2}{2(x^2+R^2)^{\frac{3}{2}}}$$

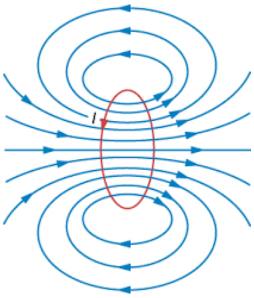
- <u>**At the centre of the loop**</u> x=0, thus,

$$B_0 = \frac{\mu_0 I}{2R}$$

- The direction of the magnetic field due to a circular coil is given by ***right-hand thumb rule***.
- Curl the palm of your right hand around the circular wire with the fingers pointing in the direction of current. Then the right hand thumb gives the direction of magnetic field.



Magnetic field lines due to a circular current loop



Relation Connecting Velocity of Light , Permittivity and Permeability

- We have

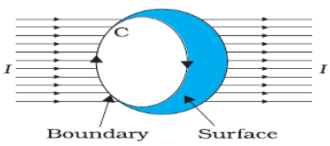
$$\varepsilon_0\mu_0 = \frac{4\pi\varepsilon_0}{1}\left(\frac{\mu_0}{4\pi}\right) = \frac{10^{-7}}{9\times10^{9}} = \frac{1}{9\times10^{16}}$$

- Thus $\varepsilon_0\mu_0 = \dfrac{1}{\left(3\times10^{8}\right)^2} = \dfrac{1}{c^2}$
- Where c – speed of light in vacuum.
- Therefore the speed of light is given by

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

- In general , $v = \dfrac{1}{\sqrt{\varepsilon\mu}}$

Ampere's Circuital Law



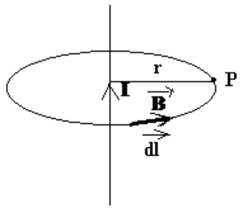
- The closed line integral of magnetic field is equal to μ_0 times the total current.

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$$

- The closed loop is called ***Amperian Loop.***

Applications Of Ampere's Circuital Law

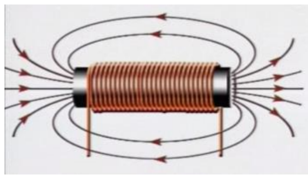
1. **Magnetic field due to a straight wire**



- Over the Amperian loop B and dl are along the same direction.
- Thus $\oint_l B.dl = \oint_l Bdl\cos 0 = B\oint_l dl$
- That is $\oint_l B.dl = B(2\pi r)$
- From ampere's circuital law , $B\times 2\pi r = \mu_0 I$
- Thus $B = \dfrac{\mu_0 I}{2\pi r}$

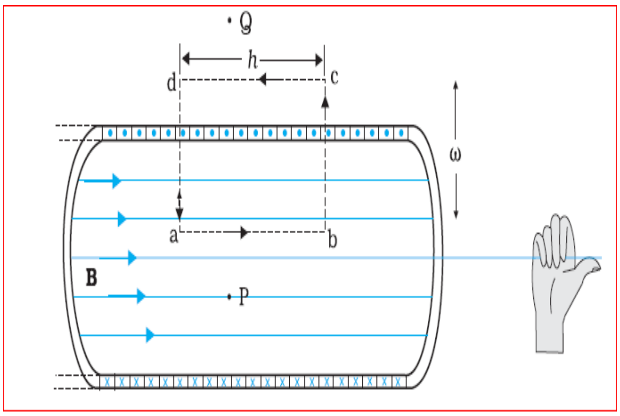
2. **Magnetic field due to a solenoid**

Solenoid



- A solenoid is an insulated copper wire closely wound in the form of a helix
- When current flows through the solenoid, it behaves as a bar magnet.
- For a long solenoid, the field outside is nearly zero.
- A solenoid is usually used to obtain a uniform magnetic field.
- If the current at one end of the solenoid is in the anticlockwise direction it will be the North Pole and if the current is in the clockwise direction it will be the South Pole.

Expression for magnetic field inside a solenoid



- Consider an amperian loop **abcda**
- The magnetic field is zero along cd,bc and da.
- The total number of turns of the solenoid is $N = nh$, where n – number of turns per unit length, h –length of the amperian loop.
- Therefore the total current enclosed by the loop is $I_e = nhI$,
- where, I –current in the solenoid
- Using Ampere's circuital law

$$\oint B.dl = Bh = \mu_0 I_e$$

$$Bh = \mu_0 nhI$$

- Therefore , the magnetic field inside the solenoid is

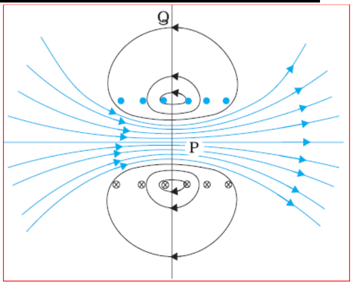
$$B = \mu_0 nI$$

- The direction of the field is given by ***Right Hand Rule.***

The magnetic field due to a solenoid can be increased by

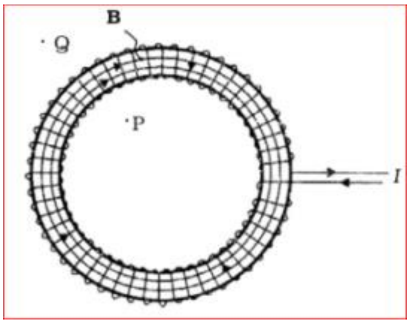
i) Increasing the no. of turns per unit length (n)
ii) Increasing the current (I)
iii) Inserting a soft iron core into the solenoid.

Magnetic Field lines of a Solenoid



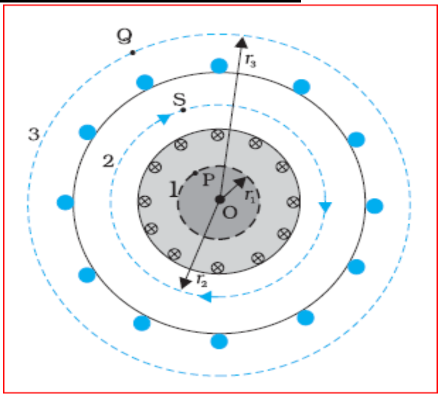
3. Magnetic Field due to a Toroid

Toroid



- Toroid is a hollow circular ring on which a large number of turns of a wire are closely wound.
- The magnetic field in the open space inside (point P) and exterior to the Toroid (point Q) is zero.
- The field B is constant inside the Toroid.

Magnetic Field due to a Toroid



For points interior (P)

- Length of the loop 1 , $L_1 = 2\pi r_1$
- The current enclosed by the loop = 0.
- Therefore

$$B_1 (2\pi r_1) = \mu_0 (0), \quad B_1 = 0$$

- Magnetic field at any point in the interior of a toroid is **zero**.

For points inside (S)

- Length of the loop , $L_2 = 2\pi r_2$
- The total current enclosed =$N I$, where N is the total number of turns and I the current.
- Applying Ampere's Circuital Law and taking $r_2 = r$

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

- Or

$$B = \mu_0 nI$$

- Where $n = \dfrac{N}{2\pi r}$

For points Exterior(Q)

- Each turn of the Toroid passes twice through the area enclosed by the Amperian Loop 3.
- For each turn current coming out of the plane of the paper is cancelled by the current going into the plane of paper.
- Therefore ***I = 0, B = 0***.

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