CHAPTER 9

RAY OPTICS AND OPTICAL INSTRUMENTS

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REFLECTION OF LIGHT

- When light is incident on a surface, it partially reflected back, partly absorbed by the surface and remaining is transmitted through the surface.
- Mirrors are used to reflect light efficiently.

Ray of Light

- The path along which a light wave travels is called ray of light.

Beam of Light

- A bundle of ray of light is called beam of light.

Angle of incidence

- The angle between the incident ray and the normal is the angle of incidence.

Angle of reflection

- The angle between the reflected ray and the normal is the angle of reflection

Spherical Mirrors

- The portion of a reflecting surface, which forms a part of a sphere, is called a spherical mirror.
- **Concave mirror** – reflecting surface towards the centre of the sphere
- **Convex mirror –** reflecting surface away from the centre of the sphere.

Some definitions

Centre of curvature (C)

- The centre of the sphere of which the mirror forms a part.

Radius of curvature (R)

- The radius of the sphere of which the mirror forms a part.

Pole

- The geometric centre of a spherical mirror is called its pole.

Principal Axis

- The line joining the pole and centre of curvature.

Aperture

- The diameter of the mirror.

Principal Focus

- The point at which, a narrow beam of light incident on the mirror parallel to its principal axis , after reflection from the mirror , meets or appears to come from.

Focal length

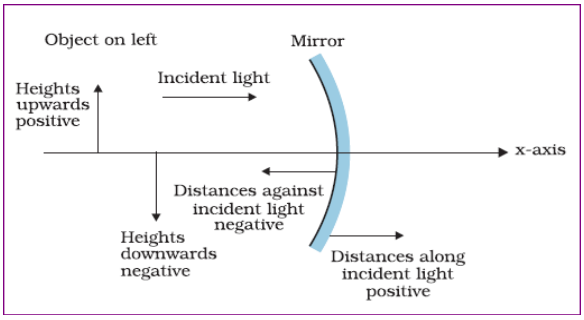
- The distance between pole and principal focus.

Spherical aberration

- The inability of a spherical mirror of large aperture to focus the marginal rays and central rays at a single point is called spherical aberration.

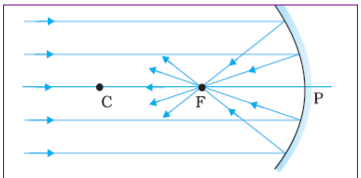
Cartesian Sign Convention

- According to this convention, all distances are measured from the pole of the mirror or the optical centre of the lens.
- The distances measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of incident light are taken as negative .
- The heights measured upwards with respect to x-axis and normal to the principal axis (x-axis) of the mirror/ lens are taken as positive).
- The heights measured downwards are taken as negative.



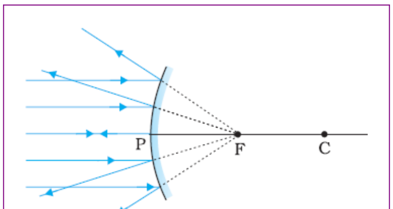
Reflection of light by spherical mirrors

Concave mirror



(a)

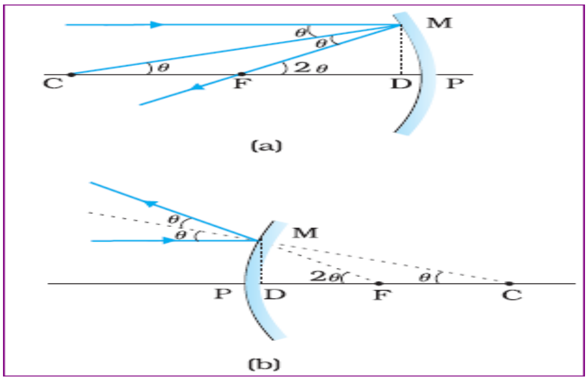
Covex Mirror



(b)

Relation between focal length and radius of curvature of a spherical mirror

- Consider a ray parallel to the principal axis striking the mirror at M.



- Thus from the diagram

$$\angle MCP = \theta \text{ and } \angle MFP = 2\theta$$

Now,

$$\tan\theta = \frac{MD}{CD} \text{ and } \tan 2\theta = \frac{MD}{FD}$$

- For small θ, tan θ ≈ θ, tan 2θ ≈ 2θ.

$$\frac{MD}{FD} = 2\,\frac{MD}{CD}$$

$$\text{or, } FD = \frac{CD}{2}$$

- For small θ, the point D is very close to the point P.
- Therefore, FD = *f and CD = R.*

$$f = R/2$$

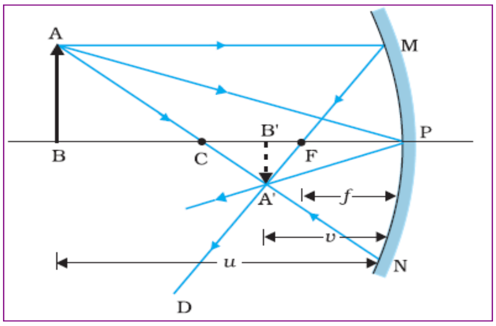
Some conventions to draw a ray diagram

- The ray from the point which is parallel to the principal axis. The reflected ray goes through the focus of the mirror.
- The ray passing through the centre of curvature of a concave mirror or appearing to pass through it for a convex mirror. The reflected ray simply retraces the path.
- The ray passing through (or directed towards) the focus of the concave mirror or appearing to pass through (or directed towards) the focus of a convex mirror. The reflected ray is parallel to the principal axis.
- The ray incident at any angle at the pole. The reflected ray follows laws of reflection.

The mirror equation

- The relation connecting the object distance (*u*), *image distance (v) and the focal length (f) is the mirror equation.*

Derivation



- In the diagram the two right-angled triangles A'B'F and MPF are similar.

- Therefore,

$$\frac{B'A'}{PM} = \frac{B'F}{FP}$$

$$\text{or } \frac{B'A'}{BA} = \frac{B'F}{FP} \quad (\because PM = AB)$$

- Since ∠ APB = ∠ A'PB', the right angled triangles A'B'P and ABP are also similar.
- Therefore,

$$\frac{B'A'}{BA} = \frac{B'P}{BP}$$

- Comparing Equations :

$$\frac{B'F}{FP} = \frac{B'P - FP}{FP} = \frac{B'P}{BP}$$

- Using sign conventions

$$B'P = -v,\ FP = -f,\ BP = -u$$

- We get

$$\frac{-v+f}{-f} = \frac{-v}{-u}$$

$$\text{or } \frac{v-f}{f} = \frac{v}{u}$$

- Therefore the mirror equation is given by

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

- The same equation can be derived for a convex mirror too.

Linear Magnification

- Linear magnification (m) is the ratio of the height of the image (h') to the height of the object (h).

$$m = \frac{h'}{h}$$

- In triangles A'B'P and ABP, we have,

$$\frac{B'A'}{BA} = \frac{B'P}{BP}$$

- With the sign convention, this becomes

$$\frac{-h'}{h} = \frac{-v}{-u}$$

so that

$$m = \frac{h'}{h} = -\frac{v}{u}$$

- Therefore the linear magnification is given by

$$m = -\frac{v}{u}$$

- The expression for magnification is same for concave and convex mirror.

Significance of magnification 'm'

- When 'm' is positive, the image is erect (virtual)
- When 'm' is negative, the image is inverted (real)
- For enlarged image, m>1
- For diminished image, m<1

Uses of spherical mirrors

Concave mirrors

- Used as reflectors of table lamps to direct light in a given area.
- Concave mirrors of large aperture are used in reflecting type astronomical telescopes.
- Shaving mirrors are made slightly concave to get erect enlarged image of the face.

Convex mirrors

- They are used in automobiles as rear view mirrors because of the two reasons:
- A convex mirror always produces an erect image.
- The image is diminished in size, so that it gives a wide field of view.

Nature of the image formed by a Concave mirror

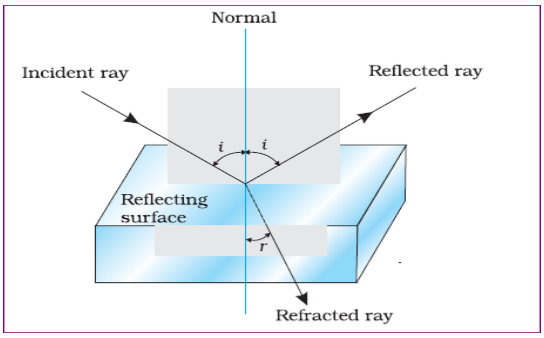
Object position	Image position	Size of image	Nature of image
At infinity	Focus (F)	Point sized	Real
Beyond C	Between F and C	Small	Real and inverted
At C	At C	Same as that of the object	Real and inverted
Between C and F	Behind C	Enlarged	Real and inverted
At F	At infinity	Highly enlarged	Real and inverted
Between F and P	Behind mirror	Enlarged	Virtual and erect

Nature of the image formed by a Convex mirror

- A convex mirror always forms a virtual and diminished image irrespective of the position of the object

REFRACTION OF LIGHT

- The phenomenon of change in path of light as it goes from one medium to another is called ***refraction.***



Laws of Refraction

- The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.

Snell's law:-

- The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant.
- Now

$$\frac{\sin i}{\sin r} = n_{21}$$

- Where **n_{21}** *is a constant, called the **refractive index** of the second medium* with respect to the first medium.

$$n_{21} = \frac{n_2}{n_1}$$

- Where n_1- absolute refractive index of the first medium and n_2 – absolute refractive index of the second medium.

Refractive index

- The refractive index of a medium depends on
  - Nature of the pair of medium
  - Wavelength of light
- Refractive index is independent of the angle of incidence**.**
- A medium having **larger value of refractive index** is called **optically denser** medium.
- A medium having **smaller value of refractive index** is called **optically rarer** medium.
- Also

$$n_{12} = \frac{1}{n_{21}}$$

- Where $n_{12} = \frac{n_1}{n_2}$
- If n_{32} *is the refractive* index of medium 3 with respect to medium 2 then

$$n_{32} = n_{31} \times n_{12}$$

- Where n_{31} is the refractive index of medium 3 with respect to medium 1.

Absolute refractive index

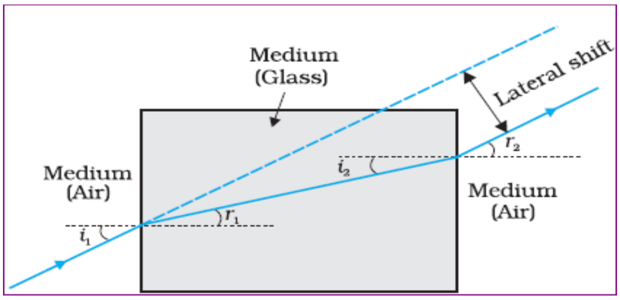
- The ratio of velocity of light in vacuum to the velocity of light in a medium is called absolute refractive index.

$$n = \frac{c}{v}$$

- Where C - velocity of light in vacuum, v- velocity of light in the medium.
- When **light enters from a rarer medium to denser medium,** the refracted ray **bends towards the normal**.
- When **light enters from a denser medium** to rarer medium, the refracted ray **bends away from the normal**.

$n_{air} = 1$	$n_{glass} = 1.5$
$n_{water} = 1.33$	$n_{diamond} = 2.42$

Refraction through a glass slab - Lateral shift

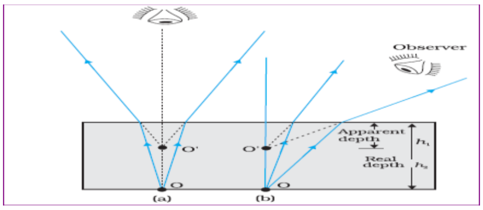


- For a rectangular slab, refraction takes place at two interfaces (air-glass and glass-air).
- When a light ray enters a glass slab it undergoes lateral displacement/ shift with respect to the incident ray.
- The perpendicular distance between the incident ray and the emergent ray, when the light is incident obliquely on a parallel sided refracting slab is called **lateral shift.**

Applications of refraction

Apparent depth

- If an object in a denser medium is viewed from a rarer medium the image appears to be raised towards the surface.
- The bottom of a tank filled with water appears to be raised due to refraction.

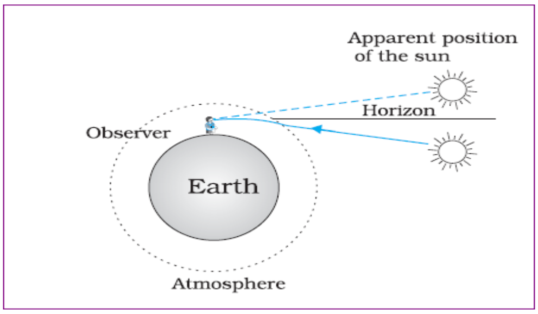


- For viewing near the normal direction

$$\text{Apparent Depth} = \frac{\text{Real Depth}}{\text{Refractive Index}}$$

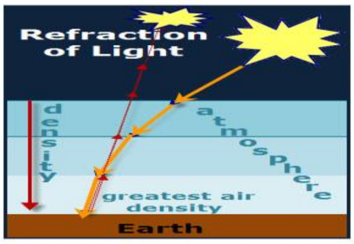
Apparent position of sun

- The sun is visible a little before the actual sunrise and until a little after the actual sunset due to refraction of light through the atmosphere.
- Time difference between actual sunset and apparent sunset is about 2 minutes.



- As we go up, the density of air in the atmosphere continuously decreases, and thus the light coming from the sun undergoes refraction.
- Thus we see the sun at an apparent position raised above the horizon.
- This is the reason for early sunrise and delayed sunset.

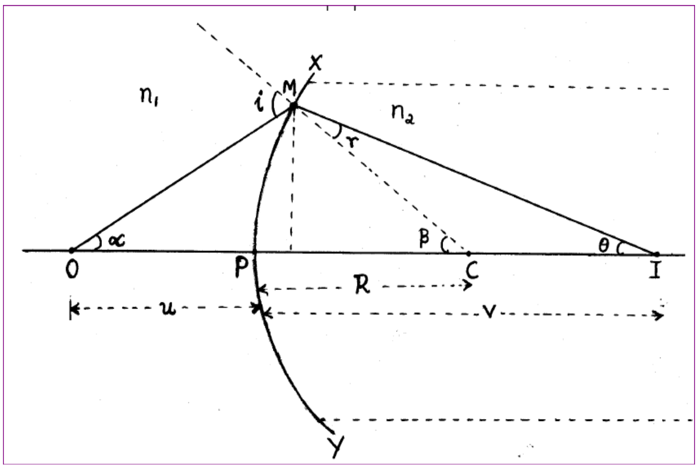
Twinkling of stars



- The light rays coming from the sun undergo refraction and hence the star is viewed at the apparent position.
- As the density of air in the atmosphere continuously changes, the apparent position also changes continuously.
- Thus the star appears to be twinkling.

REFRACTION AT SPHERICAL SURFACES

Expression for refraction at a convex surface



- For small angles , tan θ ≈ θ, thus

- From triangle OMP ,

$$\tan\alpha \approx \alpha = \frac{PM}{PO}$$

- From triangle PCM,

$$\tan\beta \approx \beta = \frac{PM}{PC}$$

- From triangle PMI,

$$\tan\theta \approx \theta = \frac{PM}{PI}$$

- From triangle OMC,
  Exterior angle = sum of interior angles
- Thus

$$i = \alpha + \beta$$
$$= \frac{PM}{PO} + \frac{PM}{PC} \ldots\ldots\ldots\ldots(1)$$

- From triangle IMC

$$\beta = r + \theta$$
$$\Rightarrow r = \beta - \theta$$
$$= \frac{PM}{PC} - \frac{PM}{PI} \ldots\ldots\ldots(2)$$

- By Snell's law

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

- If I and r are small,

$$\frac{i}{r} = \frac{n_2}{n_1}$$

$$n_1 i = n_2 r$$

- Substituting for I and r,

$$n_1\left(\frac{PM}{PO} + \frac{PM}{PC}\right) = n_2\left(\frac{PM}{PC} - \frac{PM}{PI}\right)$$

- Or

$$n_1\frac{PM}{PO} + n_1\frac{PM}{PC} = n_2\frac{PM}{PC} - n_2\frac{PM}{PI}$$

- Thus

$$\frac{n_1}{PO} + \frac{n_1}{PC} = \frac{n_2}{PC} - \frac{n_2}{PI}$$

- Therefore

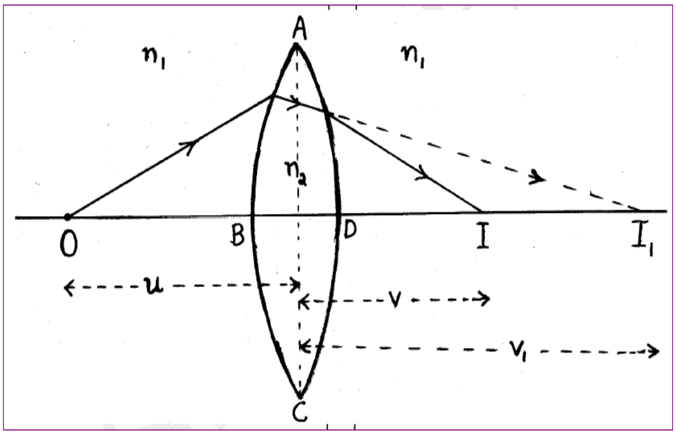
$$\frac{n_1}{PO} + \frac{n_2}{PI} = \frac{n_2 - n_1}{PC} \ldots\ldots\ldots\ldots(3)$$

- By Cartesian sign convention
  PO = -u , PI = v , PC =R
- Thus equation(3) becomes

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

- This is the **equation of refraction at convex surface.**

Refraction by a lens - Lens maker's formula



- The image formation has two steps:
  - The first refracting surface forms the image I_1 of the object O.
  - The image formed by the first refracting surface acts as the virtual object for the second refracting surface and the final image is formed at I.
- We have the curved surface formula

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

For refraction at the surface ABC

- Light ray travels from n_1 to n_2 and O is the object and I_1 is the image.
- And

$$v \to v_1, R \to R_1$$

- Here R_1 **is** the radius of curvature of ABC.

- Thus

$$\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \ldots\ldots\ldots\ldots (1)$$

For refraction at the surface ADC

- Light ray travels from n_2 to n_1.
- Here I_1 is the object and I is the image and

$$n_1 \leftrightarrow n_2, u \rightarrow v_1, v \rightarrow v, R_1 \rightarrow R_2$$

- Here **R_2 is** the radius of curvature of ADC

$$\therefore \frac{n_1}{v} - \frac{n_2}{v_1} = \frac{n_1 - n_2}{R_2} \ldots\ldots\ldots (2)$$

$$\frac{n_1}{v} - \frac{n_2}{v_1} = \frac{-(n_2 - n_1)}{R_2} \ldots\ldots\ldots (3)$$

- Adding equation 1 and 2, we get

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

- Dividing by n_1

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2 - n_1}{n_1}\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{v} - \frac{1}{u} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \ldots\ldots (4)$$

- If the object is at infinity, the image is formed at the principal focus.
- Thus if **u=∞, v=f,** equation 4 becomes

$$\frac{1}{f} - \frac{1}{\infty} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

- Thus the **lens maker's formula** is given by

$$\therefore \frac{1}{f} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \ldots\ldots\ldots\ldots (5)$$

Thin lens formula

- We have from eqn 4,

$$\frac{1}{v} - \frac{1}{u} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

- And the lens maker's formula

$$\therefore \frac{1}{f} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

- If the first medium is air n_1 = 1 and ,let n_2=n, then

$$n_{21} = \frac{n_2}{n_1} = n$$

- Thus

$$\frac{1}{f} = (n - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

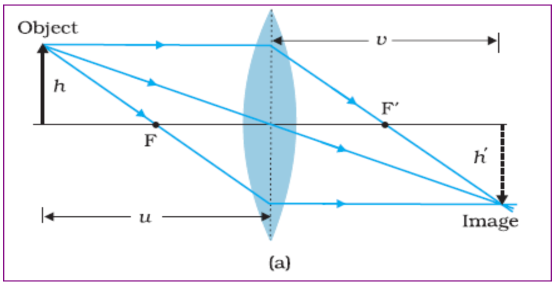
- Therefore

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

- This equation is the **thin lens formula**.
- The formula is valid for both convex as well as concave lenses and for both real and virtual images.

Linear magnification of a lens

- Magnification (*m) produced by a lens is defined,* as the ratio of the size of the image to that of the object.



$$m = \frac{h'}{h} = \frac{v}{u}$$

- The value of m is negative for real images and positive for virtual images.

Power of a lens

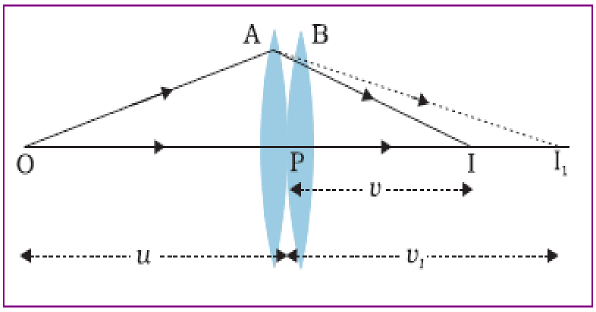
- Power of a lens is the reciprocal of focal length expressed in metre.
- Power of a lens is a measure of the convergence or divergence, which a lens introduces in the light falling on it.

$$P = \frac{1}{f}$$

- The SI unit for power of a lens is dioptre (D).

- Power of a lens is positive for a converging lens and negative for a diverging lens.

Combination of thin lenses in contact



- For the first lens, object is at O and image is at I_1.

$$u \to u,\ v \to v_1,\ f \to f_1$$

- Thus

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

- For the second lens object is I_1 and image is at I.

$$u \to v_1,\ v \to v,\ f \to f_2$$

- Therefore

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$$

- Adding Equations

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

- If the two lens-system is regarded as equivalent to a single lens of focal length *f, we have*

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

- Therefore

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

- If several thin lenses of focal length f_1, f_2, f_3,... *are in contact, the effective* focal length of their combination is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots$$

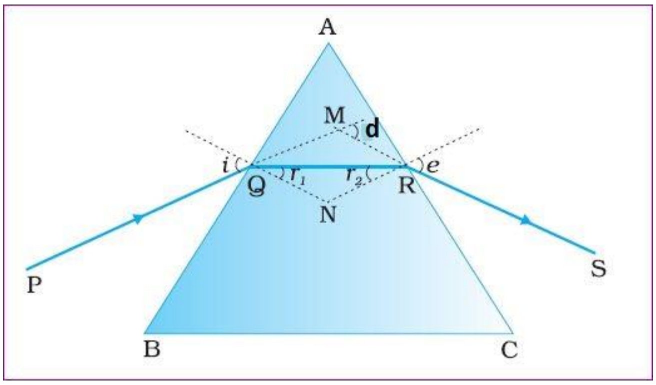
- Thus the power is given by

$$P = P_1 + P_2 + P_3 + \dots$$

- The total magnification

$$m = m_1\, m_2\, m_3\ \dots$$

REFRACTION THROUGH A PRISM



Angle of deviation, (d)

- The angle between the emergent ray RS and the direction of the incident ray PQ is called the *angle of deviation, δ.*

Angle of minimum deviation (D)

- The angle of deviation for which *the* refracted ray inside the prism becomes parallel to its base is called angle of minimum deviation.

Prism Formula (Eqn. for refractive index)

- In the quadrilateral AQNR, two of the angles (at the vertices Q and R) are right angles.
- Therefore, the sum of the other angles of the quadrilateral is 180^0.

$$\angle A + \angle QNR = 180^\circ$$

- From the triangle QNR

$$r_1 + r_2 + \angle QNR = 180^\circ$$

- Comparing these two equations

$$r_1 + r_2 = A$$

- We know ,exterior angle = sum of interior angles, thus

$$d = (i - r_1) + (e - r_2)$$

- That is

$$d = (i + e - A)$$

- Thus, the angle of deviation depends on the angle of incidence.
- At the minimum deviation, **d=D, i=e, $r_1=r_2$,** therefore

$$2r = A \text{ or } r = \frac{A}{2}$$

$$D = 2i - A, \text{ or } i = (A + D)/2$$

- Thus using Snell's law, the refractive index of the prism is given by

$$n_{21} = \frac{\sin\frac{(A+D)}{2}}{\sin\frac{A}{2}}$$

Prism formula for a small angled prism

- For a small angled prism

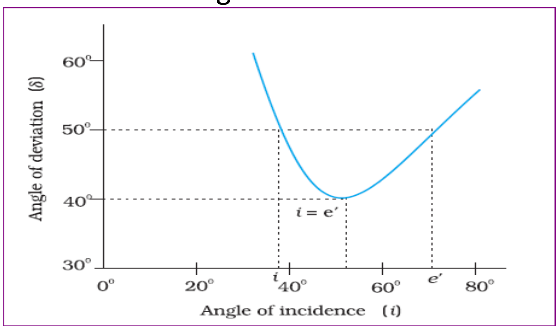
$$n_{21} = \frac{\frac{(A+D)}{2}}{\frac{A}{2}}$$

- Therefore

$$D = (n_{21} - 1)A$$

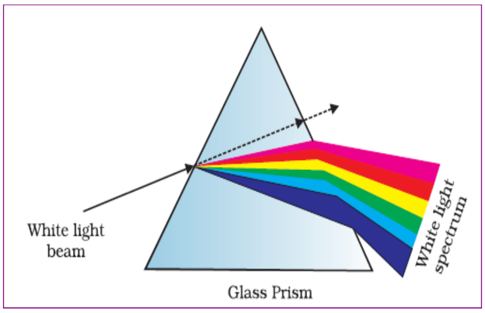
i-d curve

- It is the plot between the angle of deviation and angle of incidence.

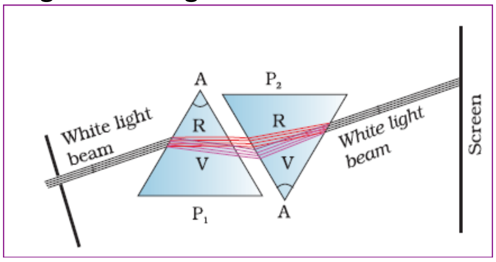


DISPERSION BY A PRISM

- The phenomenon of splitting of light into its component colours is known as *dispersion.*
- *The pattern of colour components* of light is called the spectrum of light.



- Thick lenses could be assumed as made of many prisms, therefore, thick lenses show ***chromatic aberration*** *due to* dispersion of light.
- When white light is passed through a prism, it splits into its seven component colors (**VIBGYOR).**
- If we place a second prism in an inverted position, close to the first prism, the second prism recombines the colors and we get white light.



Cause of dispersion

- Dispersion takes place because the refractive index of medium for different wavelengths (colors) is different.

Dispersive medium

- The medium in which the different colours of light travel with different velocities is called a dispersive medium.
- Eg :- Glass

Non-Dispersive medium

- The medium in which all colours travel with the same speed is called non-dispersive medium.
  Eg:- vacuum

Chromatic abberation

- The inability of a lens to focus all wavelength to a single point is called chromatic aberration.

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