



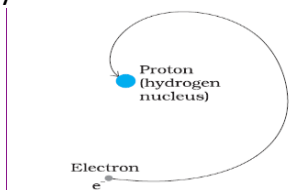
## Chapter Twelve

### ATOMS

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#### Limitations of Rutherford Model

- Rutherford's model fails to account for the stability of the atom.
- The energy of an accelerating electron should continuously decrease and the electron would spiral inward and eventually fall into the nucleus.



- Rutherford's model does not explain the line spectra of atoms.

#### BOHR MODEL OF THE HYDROGEN ATOM

- Bohr combined classical and early quantum concepts and gave his theory in the form of three postulates:

##### Postulate I

- Electrons in an atom can revolve in certain stable orbits without radiating energy.**
- According to this postulate, each atom has certain definite stable states in which it can exist, and each possible state has definite total energy.
- These are called the **stationary states** of the atom

##### Postulate II

- The electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of  $h/2\pi$  where  $h$  is the Planck's constant ( $= 6.6 \times 10^{-34}$  J s).**
- Thus the angular momentum ( $L$ ) of the orbiting electron is quantised.
- That is  $L = nh/2\pi$ , where  $n = 1, 2, 3, \dots$ , is the principal quantum number.

##### Postulate III

- An electron might make a transition from one of its specified non-radiating orbits to another of lower energy.**

- A photon is emitted having energy equal to the energy difference between the initial and final states.
- The frequency of the emitted photon is then given by

$$h\nu = E_i - E_f$$

#### Radii of Bohr's Stationary orbits:

- The centripetal force for the revolution of electrons round the nucleus is provided by the electrostatic force of attraction between the nucleus and the electron.
- Thus

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

- Therefore

$$mv^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

- From Bohr's II postulate the angular momentum  $L = mvr$ , is given by

$$mvr = \frac{nh}{2\pi}$$

- Thus

$$v = \frac{nh}{2\pi mr}$$

- Therefore

$$m \left( \frac{nh}{2\pi mr} \right)^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad m \frac{n^2 h^2}{4\pi^2 m^2 r^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

- That is

$$\frac{n^2 h^2}{\pi m r} = \frac{e^2}{\epsilon_0}$$

- The radius is given by

$$r = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}$$

- In general

$$r_n = \left( \frac{\epsilon_0 h^2}{\pi m e^2} \right) n^2$$

- Thus

$$r_n \propto n^2$$

- The radii of the stationary orbits are in the ratio,  $1^2:2^2:3^2:\dots$  or  $1:4:9:\dots$

- The stationary orbits are not equally spaced.

### Bohr Radius

- The radius of the lowest orbit ( $n=1$ ) is called Bohr radius.
- The Bohr radius is given by

$$a_0 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

- Substituting the values we get

$$a_0 = 5.29 \times 10^{-11} \text{ m}$$

- Thus the radius of  $n^{\text{th}}$  orbit becomes:

$$r_n = a_0 n^2$$

### Velocity of electrons in an orbit

- We have

$$v = \frac{nh}{2\pi m r}$$

- But

$$r = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}$$

- Therefore

$$v = \frac{nh}{2\pi m \left( \frac{\epsilon_0 h^2}{\pi m e^2} \right) n^2}$$

- That is

$$v = \frac{e^2}{2\epsilon_0 n h}$$

- In general

$$v_n = \frac{e^2}{2\epsilon_0 n h}$$



$$K = \frac{1}{2} m v^2 = \frac{e^2}{8\pi\epsilon_0 r}$$

### Potential energy

- The electrostatic potential energy of an orbital electron is given by

$$U = -\frac{e^2}{4\pi\epsilon_0 r}$$

### Total energy

- Total energy is given by

$$E = K + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r}$$

- That is

$$E = \frac{-e^2}{8\pi\epsilon_0 r}$$

- But, we have

$$r = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}$$

- Thus

$$E = \frac{-e^2}{8\pi\epsilon_0 \times \frac{\epsilon_0 h^2 n^2}{\pi m e^2}}$$

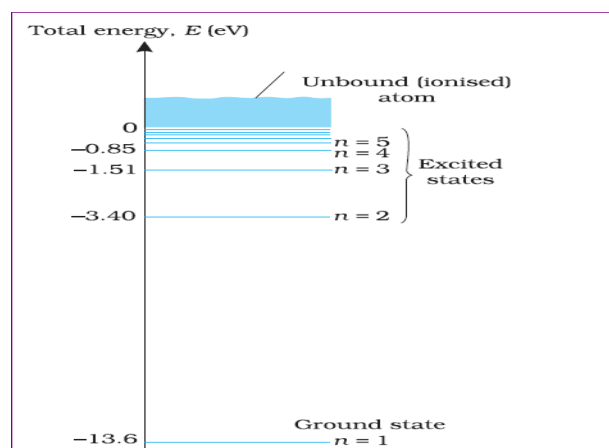
- In general

$$E_n = \frac{-m e^4}{8\epsilon_0^2 h^2 n^2}$$

- Substituting the values we get

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

### Energy level diagram of hydrogen atom



### Total energy of an orbiting electron

#### Kinetic energy

- For an orbiting electron, we have

$$\frac{m v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

- Or

$$m v^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

- Thus the kinetic energy is given by

### Excitation energy

- Excitation energy is the **energy required to excite an electron from its ground state to an excited state.**
- First excitation energy of hydrogen atom required to excite the electron from  $n = 1$  to  $n = 2$  orbit of hydrogen atom. That is  $(-3.4) - (-13.6) = 10.2 \text{ eV}$ .

### Excitation potential

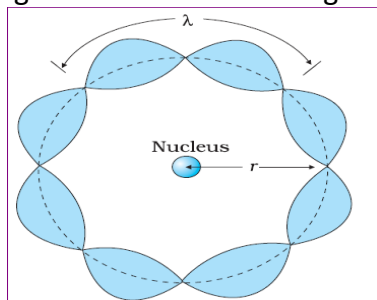
- Excitation potential of an excited state is the potential difference through which electron in an atom has to be accelerated so as to excite it from its ground state to the given excited state.
- The **first excitation potential of H atom** is 10.2V.

### Ionization energy

- Ionisation energy is the energy required to take an electron completely out of the atom.
- The ionization energy of hydrogen atom is 13.6 eV.

### DE BROGLIE'S EXPLANATION OF BOHR'S SECOND POSTULATE OF QUANTISATION

- Louis de Broglie argued that the electron in its circular orbit, as proposed by Bohr, must be seen as a particle wave.
- In analogy to waves travelling on a string, particle waves too can lead to standing waves under resonant conditions.
- In a string, standing waves are formed when the total distance travelled by a wave down the string and back is one wavelength, two wavelengths, or any integral number of wavelengths.



- For an electron moving in  $n$ th circular orbit of radius  $r_n$ , the total distance is the circumference of the orbit,  $2\pi r_n$ .

$$2\pi r_n = n\lambda, \quad n = 1, 2, 3\dots$$

- But we have  $\lambda = h/p$ ,

$$2\pi r_n = n h/mv_n \quad \text{or} \quad m v_n r_n = nh/2\pi$$

- This is the quantum condition proposed by Bohr for the angular momentum of the electron

### Limitations of Bohr Model

- Bohr's theory is applicable only to single electron atoms.
- This theory gives no idea about relative intensities of spectral lines.
- Could not explain the fine structure of hydrogen spectrum.

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