Chapter Thirteen
NUCLEI

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RADIOACTIVITY

- **H. Becquerel** discovered radioactivity in 1896.
- Radioactivity is a nuclear phenomenon in which an unstable nucleus undergoes a decay.This is referred to as ***radioactive decay***.
- ***Three types of radioactive decay* occur in nature** :
- **α-decay** in which a helium nucleus (He) is emitted;
- **β-decay** in which electrons or positrons (particles with the same mass as electrons, but with a charge exactly opposite to that of electron) are emitted;
- **γ-decay** in which high energy (hundreds of keV or more) photons are emitted.

Law of radioactive decay

- ***This law states that the number of nuclei undergoing the decay per unit time is proportional to the total number of nuclei in the sample.***
- **I**f a sample contains N undecayed nuclei and let dN nuclei disintegrate in dt second, thus the rate of disintegration

$$\frac{dN}{dt} \alpha - N$$

- The negative sign shows that the number of nuclei decreases with time.
- Thus

$$\boxed{\frac{dN}{dt} = -\lambda N}$$

- Where **λ** is called the **radioactive *decay constant or disintegration constant*.**

$$\boxed{\text{or,} \quad \frac{dN}{N} = -\lambda dt}$$

- Now, integrating both sides of the above equation, we get

$$\boxed{\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt}$$

$$\boxed{\text{or,} \ \ln N - \ln N_0 = -\lambda (t - t_0)}$$

- Here N_0 is the number of radioactive nuclei in the sample at some arbitrary time t_0 and N is the number of radioactive nuclei at any subsequent time t.
- Setting $t_0 = 0$

$$\boxed{\ln \frac{N}{N_0} = -\lambda t}$$

- Thus $\mathbf{N = N_0 e^{-\lambda t}}$

Decay Rate

- It gives the number of nuclei decaying per unit time

$$\boxed{R = -\frac{dN}{dt}}$$

$$\boxed{R = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}}$$

$$\boxed{\text{or,} \quad R = R_0 e^{-\lambda t}}$$

- Here R_0 *is the radioactive decay* rate at time *t = 0, and R is the rate at any subsequent* time *t.*
- Thus

$$\boxed{R = \lambda N}$$

- The **total decay rate R** *of a sample of one or more radionuclide's* is called ***the activity*** *of that sample.*
- *The **SI unit for activity is** **becquerel***, named after the discoverer of radioactivity.
- **1 becquerel = 1Bq = 1 decay per second**
- An older unit, the **curie**, is still in common use.

$$\boxed{1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq (decays per second)}}$$

Half life period ($T_{1/2}$)

- **It is the time in which the number of undecayed nuclei falls into half of its original number.**
- Thus it is the time at which both N and R have been reduced to one-half their initial values.

$$\boxed{\begin{aligned} T_{1/2} &= \frac{\ln 2}{\lambda} \\ &= \frac{0.693}{\lambda} \end{aligned}}$$

We have $N = N_0 e^{-\lambda t}$

When $t = T_{1/2}$, $N = \frac{N_0}{2}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

i.e. $e^{-\lambda T_{1/2}} = 2$

or $\lambda T_{1/2} = \ln 2$

$$\boxed{T_{1/2} = \frac{0.693}{\lambda}}$$

Mean life (τ)

- It is the average life of all the nuclei in a radioactive sample.
- Mean life = total life time of all nuclei / total number of nuclei present initially

$$\tau = \frac{1}{\lambda}$$

- The number of nuclei which decay in the time interval *t to t + Δt* is

$$\boxed{R(t)\Delta t = (\lambda N_0 e^{-\lambda t}\Delta t)}$$

- Each of them has lived for time *t*. Thus the total life of all these nuclei would be

$$\boxed{t\,\lambda N_0 e^{-\lambda t}\,\Delta t}$$

- Therefore mean life is given by

$$\tau = \frac{\lambda N_0 \int_0^\infty t e^{-\lambda t} dt}{N_0} = \lambda \int_0^\infty t e^{-\lambda t} dt$$

One can show by performing this integral that $\tau = 1/\lambda$

We summarise these results with the following:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2 \qquad (1$$

Alpha decay

- When a nucleus undergoes *alpha-decay, it transforms to a different* nucleus by emitting an alpha-particle (a helium nucleus)

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}$$

- The difference between the initial mass energy and the final mass energy of the decay products is called the ***<u>Q value of the process or the disintegration energy</u>***.

$$\boxed{Q = (m_X - m_Y - m_{He})\,c^2}$$

- This energy is shared by the daughter nucleus and the alpha particle,in the form of kinetic energy
- Alpha-decay obeys the radioactive law
- Alpha particles are positively charged particles
- Can be deflected by electric and magnetic fields.
- Can affect photographic plates.

Beta decay

- **A nucleus that decays spontaneously by emitting an electron or a positron is said to undergo *beta decay.***
- **<u>In beta-minus decay</u>**, a neutron transforms into a proton within the nucleus according to

$$\boxed{n \rightarrow p + e^- + \bar{\nu}}$$

  - Where ν is the antineutrino
- In beta minus (β^-) decay, an electron is emitted by the nucleus.
- **Eg:**

$$^{32}_{15}\mathrm{P} \rightarrow {}^{32}_{16}\mathrm{S} + e^- + \bar{\nu} \qquad (T_{1/2} = 14.3\ \mathrm{d})$$

- When β^- particles are emitted, **<u>the atomic number increases by one.</u>**
- **<u>In beta-plus decay</u>**, a proton transforms into neutron (inside the nucleus)

$$\boxed{p \rightarrow n + e^+ + \nu}$$

- Where ν is the neutrino
- In beta plus (β+) decay, a positron is emitted by the nucleus,
- **Eg:**

$$^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + e^+ + \nu \quad (T_{1/2} = 2.6\ \mathrm{y})$$

- When β^+ particles are emitted the **<u>atomic number decreases by one.</u>**

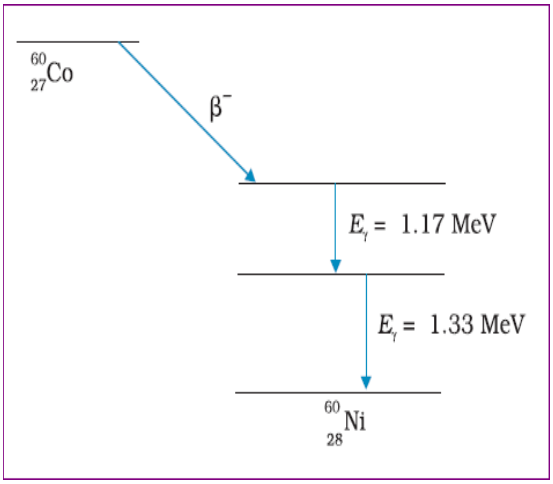
Neutrinos and Antineutrinos

- The particles which are emitted from the nucleus along with the electron or positron during the decay process.

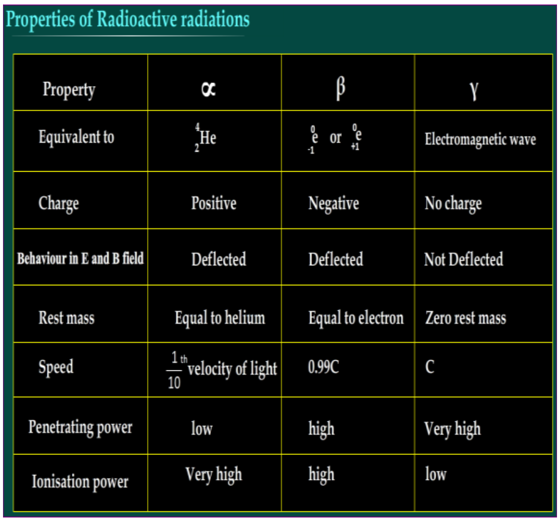
- Neutrinos interact only very weakly with matter; they can even penetrate the earth without being absorbed.

Gamma decay

- There are energy levels in a nucleus, just like there are energy levels in atoms.
- When a nucleus is in an excited state, it can make a transition to a lower energy state by the emission of electromagnetic radiation.
- As the energy differences between levels in a nucleus are of the order of MeV, the photons emitted by the nuclei have MeV energies and are called gamma rays.



- Most radionuclides after an alpha decay or a beta decay leave the daughter nucleus in an excited state.
- The daughter nucleus reaches the ground state by a single transition or sometimes by successive transitions by emitting one or more gamma rays.



Properties of Radioactive radiations

Property	∝	β	γ
Equivalent to	^4_2He	$^0_{-1}e$ or $^0_{+1}e$	Electromagnetic wave
Charge	Positive	Negative	No charge
Behaviour in E and B field	Deflected	Deflected	Not Deflected
Rest mass	Equal to helium	Equal to electron	Zero rest mass
Speed	$\frac{1}{10}^{th}$ velocity of light	0.99C	C
Penetrating power	low	high	Very high
Ionisation power	Very high	high	low

NUCLEAR ENERGY

- In conventional energy sources like coal or petroleum, energy is released through chemical reactions.
- One kilogram of coal on burning gives 10^7 J of energy, whereas 1 kg of uranium, which undergoes fission, will generate on fission 10^{14} J of energy.

Nuclear Fission

- Enrico Fermi found that when neutrons bombard various elements, new radioactive elements are produced.
- Eg:

$$^1_0n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3\,^1_0n$$

$$^1_0n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{133}_{51}Sb + ^{99}_{41}Nb + 4\,^1_0n$$

Still another example is

$$^1_0n + ^{235}_{92}U \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^1_0n$$

- The fragment nuclei produced in fission are highly neutron-rich and unstable.
- They are radioactive and emit beta particles in succession until each reaches a stable end product.
- The **energy released (the *Q value*) in the *fission reaction of nuclei like* uranium** is of the order of **200 MeV** per fissioning nucleus.
- The disintegration energy in fission events first appears as the kinetic energy of the fragments and neutrons.
- Eventually it is transferred to the surrounding matter appearing as heat.
- The source of energy in **nuclear reactors, which produce electricity, is nuclear fission**.
- The enormous energy released in an **atom bomb comes from uncontrolled nuclear** fission.

Nuclear reactor

- Neutrons liberated in fission of a uranium nucleus were so energetic that they would escape instead of triggering another fission reaction.

- Slow neutrons have a much higher intrinsic probability of inducing fission in U (235) than fast neutrons.
- The **average energy of a neutron produced in fission of U (235) is 2 MeV**.
- In reactors, light nuclei called ***moderators*** *are provided along with the fissionable nuclei for* ***slowing*** **down fast neutrons**.
- The moderators commonly used are **water, heavy water (D2O) and graphite**.
- The Apsara reactor at the Bhabha Atomic Research Centre (BARC), Mumbai, uses water as moderator.
- The other Indian reactors, which are used for power production, use heavy water as moderator.

Multiplication factor

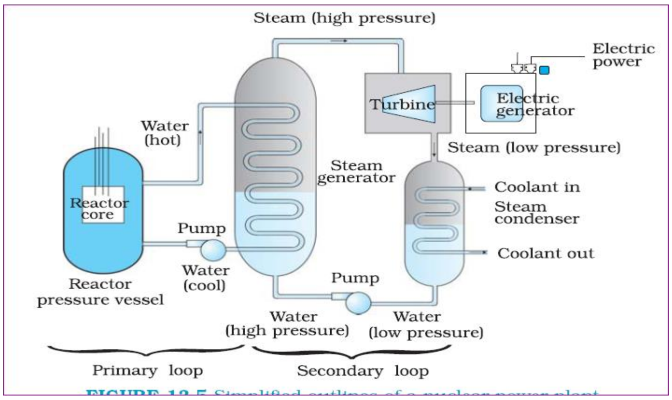
- It is the ratio of number of fission produced by a given generation of neutrons to the number of fission of the preceding generation.
- It is the measure of the growth rate of the neutrons in the reactor.
- For *K = 1, the operation of the reactor* is said to be *critical, which is what we wish it to be for steady power* operation.
- If *K becomes greater than one, the reaction rate and the reactor* power increases exponentially.
- Unless the factor *K is brought down very* close to unity, the reactor will become supercritical and can even explode.
- The explosion of the Chernobyl reactor in Ukraine in 1986 is a sad reminder that accidents in a nuclear reactor can be catastrophic.
- The reaction rate is controlled through **control-rods** made out of neutron-absorbing material such as **cadmium.**
- In addition to control rods, reactors are provided with ***safety rods*** which, when required, can be inserted into the reactor and K can be reduced rapidly to less than unity.
- The abundant U(238) isotope, which does not fission, on capturing a neutron leads to the formation of plutonium.

$$^{238}_{92}\mathrm{U} + \mathrm{n} \rightarrow {}^{239}_{92}\mathrm{U} \rightarrow {}^{239}_{93}\mathrm{Np} + e^{-} + \overline{\nu}$$

$$^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + e^{-} + \overline{\nu}$$

- Plutonium is highly radioactive and can also undergo fission under bombardment by slow neutrons

Pressurized-water reactor



- In such a reactor, water is used both as the moderator and as the heat transfer medium
- In the *primary-loop, water is circulated through the reactor vessel and transfers* energy at high temperature and pressure (at about 600 K and 150 atm) to the steam generator, which is part of the *secondary-loop.*
- *In the steam* generator, evaporation provides high-pressure steam to operate the turbine that drives the electric generator.
- The low-pressure steam from the turbine is cooled and condensed to water and forced back into the steam generator.
- A kilogram of U(235) on complete fission generates about 3×10^{4} MW.
- in nuclear reactions highly radioactive elements are continuously produced.
- Therefore, an unavoidable feature of reactor operation is the accumulation of radioactive waste, including both fission products and heavy *transuranic elements such as plutonium and* americium.

Nuclear fusion

- Energy can be released if two light nuclei combine to form a single larger nucleus, a process called *nuclear fusion.*

$$ {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu + 0.42\ \mathrm{MeV} $$

$$ {}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + n + 3.27\ \mathrm{MeV} $$

$$ {}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} + 4.03\ \mathrm{MeV} $$

- The fusion reaction in the sun is a multi-step process in which hydrogen is burned into helium, hydrogen being the ‘fuel’ and helium the ‘ashes’.
- The ***proton-proton (p, p) cycle by which this occurs is*** **represented** by the following sets of reactions:.

$$ {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu + 0.42\ \mathrm{MeV} \qquad \text{(i)} $$

$$ e^{+} + e^{-} \rightarrow \gamma + \gamma + 1.02\ \mathrm{MeV} \qquad \text{(ii)} $$

$$ {}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma + 5.49\ \mathrm{MeV} \qquad \text{(iii)} $$

$$ {}^{3}_{2}\mathrm{H} + {}^{3}_{2}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} + 12.86\ \mathrm{MeV} \qquad \text{(iv)} $$

- The combined reaction is

$$ 4\,{}^{1}_{1}\mathrm{H} + 2e^{-} \rightarrow {}^{4}_{2}\mathrm{He} + 2\nu + 6\gamma + 26.7\,\mathrm{MeV} $$

$$ \text{or}\ \ (4\,{}^{1}_{1}\mathrm{H} + 4e^{-}) \rightarrow ({}^{4}_{2}\mathrm{He} + 2e^{-}) + 2\nu + 6\gamma + 26.7\,\mathrm{MeV} $$

- In sun it has been going on for about 5 × 10^{9} y, and calculations show that there is enough hydrogen to keep the sun going for about the same time into the future.
- In about 5 billion years, however, the sun’s core, which by that time will be largely helium, will begin to cool and the sun will start to collapse under its own gravity.
- This will raise the core temperature and cause the outer envelope to expand, turning the sun into what is called a ***red giant***.
- If the core temperature increases to 10^{8} K again, energy can be produced through fusion once more – this time by burning helium to make carbon.

Controlled thermonuclear fusion

- The first thermonuclear reaction on earth occurred at Eniwetok Atoll on November 1, 1952, when USA exploded a fusion device, generating energy equivalent to 10 million tons of TNT (one ton of TNT on explosion releases 2.6 × 10’22 MeV of energy).
- A sustained and controllable source of fusion power is considerably more difficult to achieve.

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