TEXTBOOK EXERCISES 2.3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}.

Soln: (i) Let R_1 = {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}. Since the first elements 2, 5, 8, 11, 14 and 17 having unique images, R_1 is a function with domain = {2, 5, 8, 11, 14, 17} and Range = {1}.

(ii) Let R_2 = {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

Since the first elements 2, 4, 6, 8, 10, 12, 14 having unique images, R_2 is a function with domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)} is not a function because 1 has two images.

2. Find the domain and range of the following real functions:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9-x^2}$.

Soln: (i) $f(x) = -|x|$ is from $R \to R$.

$\therefore$ Domain of f = R

We know that $|x| \geq 0, \forall x \in R$. $\therefore -|x| \leq 0$.

$\therefore$ Range of f = $(-\infty, 0]$ (Set of non-positive reals)

(ii) Given function $f(x) = \sqrt{9-x^2}$ is a real function.

f(x) is defined for all real values of x for which $9 - x^2 \geq 0$ (if $9 - x^2 < 0$, f (x) is an imaginary number.) $\Rightarrow x^2 \leq 9 \Rightarrow -3 \leq x \leq 3$

$\therefore$ Domain of f = [–3, 3].

Let $y = f(x) = \sqrt{9-x^2} \Rightarrow y^2 = 9 - x^2$

$\Rightarrow x^2 = 9 - y^2 \Rightarrow 9 - y^2 \geq 0 \Rightarrow y^2 \leq 9 \Rightarrow -3 \leq y \leq 3$.

But $y = \sqrt{9-x^2} \geq 0 \therefore 0 \leq y \leq 3$

$\therefore$ Range of f = [0, 3]

3. A function f is defined by $f(x) = 2x - 5$. Write down the values of (i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$.

Soln: Given f (x) = 2x – 5.

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2 \times -3 - 5 = -6 - 5 = -11$

4. The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C, when $t(C) = 212$.

Soln: Given $t(C) = \frac{9C}{5} + 32$

(i) $t(0) = \frac{9}{5} \times 0 + 32 = 0 + 32 = 32$

(ii) $t(28) = \frac{9 \times 28}{5} + 32 = 50.4 + 32 = 82.4$

(iii) $t(-10) = \frac{9}{5} \times -10 + 32 = -18 + 32 = 14$

(iv) Given $t(C) = 212 \Rightarrow 212 = \frac{9C}{5} + 32 \Rightarrow \frac{9C}{5} = 180$

$\Rightarrow C = \frac{5}{9} \times 180 = 5 \times 20 = 100$

5. Find the range of each of the following functions.

(i) $f(x) = 2 - 3x, x \in R, x > 0$.

(ii) $f(x) = x^2 + 2$, x is a real number.

(iii) $f(x) = x$, x is a real number.

Soln: (i) Given $f(x) = 2 - 3x, x \in R, x > 0$

Since $x > 0, -3x < 0 \therefore 2 - 3x < 2 - 0 \Rightarrow f(x) < 2$

$\Rightarrow R_f$ is the set of all reals $< 2 \Rightarrow R_f = (-\infty, 2)$.

(ii) Given $f(x) = x^2 + 2$, x is real.

Since $x^2 \geq 0, x^2 + 2 \geq 0 + 2 \Rightarrow f(x) \geq 2$.

$\therefore$ Range of f is the set of all reals ≥ 2.

$\Rightarrow R_f = [2, \infty)$

(iii) Given f(x) = x, x is a real number.

Since x is a real number, f(x) is also a real number.

$\therefore R_f$ = set of all real numbers = R.