Relations and Functions
Focus area class-1

Types of functions

① One-one function [Injective function]

A function $f: A \longrightarrow B$ is said to be a one-one function if distinct elements of A has distinct images in B. Otherwise the function is said to be many-one function.

Note

To Prove a function f is 1-1, we have to show that

$$f(x) = f(y) \Longrightarrow x = y$$

② Onto function [surjective function]

A function $f: A \longrightarrow B$ is said to be onto if each element of B is the image of some element of A under f.

ie range of f = co-domain of f.

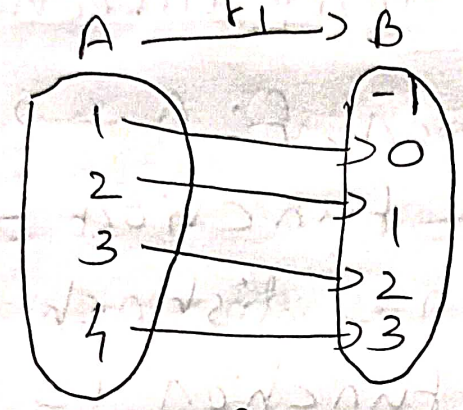
Otherwise the function is said to be an into function.

Note: To Prove a function $f: A \longrightarrow B$ is onto, take some element $y \in B$, there exist an $x \in A$ such that $f(x) = y$.

③ Bijective function

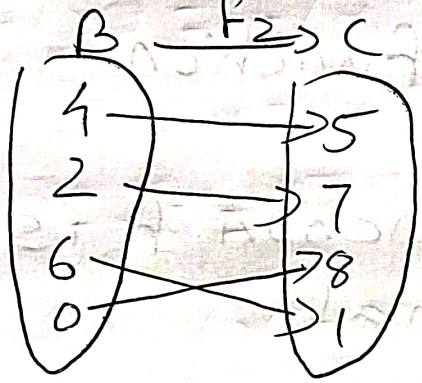
A function which is both 1-1 and onto is called a bijective function.

eg ①



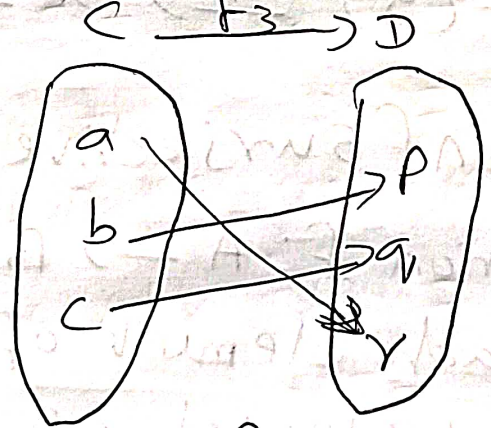
one-one into not bijective.

eg ②



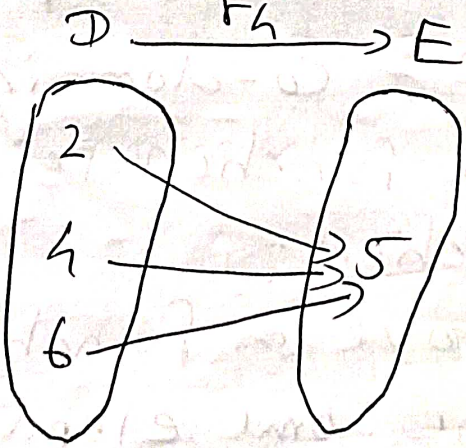
one-one onto bijective

eg: ③



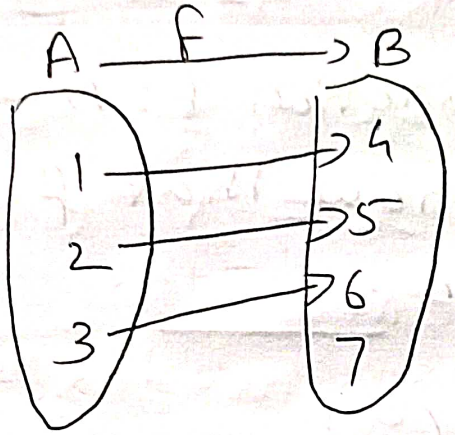
one-one onto bijective

eg: ④



many one onto not bijective

① Consider a function $f: A \to B$ as $f = \{(1,4), (2,5), (3,6)\}$ where $A = \{1,2,3\}$, $B = \{4,5,6,7\}$. check whether f is 1-1 or onto.



clearly f is 1-1, since every element has distinct images.

f is not onto, since 7 has no Pre-image.

② Consider the function $f: R \to R$ given by $f(x) = x^2$. check whether f is 1-1 or onto.

Let $f(x) = f(y)$

$$x^2 = y^2$$

$$x^2 - y^2 = 0$$

$$(x+y)(x-y) = 0$$

$$x+y = 0 \quad \text{or} \quad x-y = 0$$

$$x = -y \quad \text{or} \quad x = y$$

$\therefore$ f is not one-one.

Let $y \in R$, $f(x) = y$

$$x^2 = y$$

$$x = \sqrt{y} \notin R$$

since $\sqrt{-3} \notin R$

$\therefore$ f is not onto.

(3) Consider a function $f: N \to N$ given by $f(x) = x^2$. Show that f is 1-1 and f not onto.

Let $f(x) = f(y)$

$$x^2 = y^2$$

$$x^2 - y^2 = 0$$

$$(x+y)(x-y) = 0$$

$x + y \neq 0$, $x - y = 0$

$$x = y$$

$\therefore$ f is one-one.

Let $y \in N$, $f(x) = y$

$$x^2 = y$$

$$x = \sqrt{y} \notin N \quad \sqrt{2} \notin N$$

$\therefore$ f is not onto.

(4) Show that $f: N \to N$ given by $f(x) = 3x$ is 1-1 but not onto

Let $f(x) = f(y)$

$3x = 3y$

$x = y$

$\therefore f$ is one-one

Let $y \in N$, $f(x) = y$

$3x = y$

$x = y/3 \notin N$ $\quad 1/3 \notin N$

$\therefore f$ is not onto

(5) Show that $f: R \to R$ given by $f(x) = 3x$ is bijective.

Let $f(x) = f(y)$

$3x = 3y$

$x = y$

$\therefore f$ is one-one

Let $y \in R$, $f(x) = y$

$3x = y$

$x = y/3 \in R$

$\therefore f$ is onto

Since f is both one-one and onto

f is a bijection.