

Relations and Functions

Focus area class-2

1, Show that $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 3 - 4x$, is bijjective.

$$\text{Let } f(x) = f(y)$$

$$3 - 4x = 3 - 4y$$

$$-4x = -4y$$

$$x = y$$

$\therefore f$ is one-one.

$$\text{Let } y \in \mathbb{R}, f(x) = y$$

$$3 - 4x = y$$

$$-4x = y - 3$$

$$x = \frac{y - 3}{-4} \in \mathbb{R}$$

$\therefore f$ is onto.

Since f is both one-one and onto it is a bijjective function.

② Show that $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3$, is bijjective.

$$\text{Let } f(x) = f(y)$$

$$x^3 = y^3$$

$$x^3 - y^3 = 0$$

$$(x - y)(x^2 + xy + y^2) = 0$$

$$x^2 + xy + y^2 \neq 0, \quad x - y = 0$$

$$\therefore x = y$$

$\therefore f$ is one-one.

Let $y \in \mathbb{R}$, $f(x) = y$

$$x^3 = y$$

$$x = y^{1/3} \in \mathbb{R}$$

$\therefore f$ is onto

Since f is both one-one and onto it is bijective.

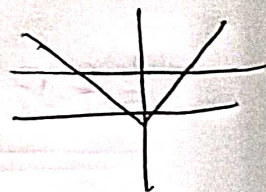
(3) Consider $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x|$
show that f is neither 1-1 nor onto.

$$f(1) = 1$$

$$f(-1) = 1$$

$\therefore f(x)$ is not one-one

Negative real numbers has no pre-image. Hence f is not onto.



(4) Let $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(n) = \begin{cases} \frac{n+1}{2}, & n \text{ odd} \\ \frac{n}{2}, & n \text{ even} \end{cases}$
show that f is not 1-1

$$f(1) = \frac{1+1}{2} = 1, \quad f(2) = \frac{2}{2} = 1$$

$\therefore f$ is not 1-1.