Relations and Functions

Focus area class - 3

Composition of functions

Let $f : A \longrightarrow B$, $g : B \longrightarrow C$ be two functions. Then the composition of these functions denoted by $g \circ f$ is a function from $A \longrightarrow C$ such that

$$[g \circ f](x) = g[f(x)], \quad \forall x \in A$$

Also $[f \circ g](x) = f[g(x)]$

$$[f \circ f](x) = f[f(x)],$$

① Let $f(x) = \cos x$, $g(x) = 3x^2$

Show that $f \circ g \neq g \circ f$.

$$f \circ g(x) = f[g(x)]$$
$$= f[3x^2] = \cos(3x^2)$$

$$g \circ f(x) = g[f(x)]$$
$$= g[\cos x] = 3(\cos x)^2$$
$$= 3\cos^2 x$$

$\therefore f \circ g \neq g \circ f$

② Let $f(x) = 8x^3$, $g(x) = x^{1/3}$

Find fog and gof.

$$fog(x) = f[g(x)]$$
$$= f[x^{1/3}]$$
$$= 8(x^{1/3})^3 = \underline{\underline{8x}}$$

$$gof(x) = g[f(x)] = g[8x^3]$$
$$= (8x^3)^{1/3} = 8^{1/3}.(x^3)^{1/3}$$
$$= \underline{\underline{2x}}$$

③ Let $f(x) = |x|$, $g(x) = |5x-2|$

Find fog and gof.

$$fog(x) = f[g(x)]$$
$$= f[|5x-2|]$$
$$= ||5x-2|| = \underline{\underline{|5x-2|}}$$

$$gof(x) = g[f(x)]$$
$$= g[|x|]$$
$$= \underline{\underline{|5|x|-2|}}$$

④ Let $f : \{1,3,4\} \longrightarrow \{1,2,5\}$ and
$g : \{1,2,5\} \longrightarrow \{1,3\}$ given by
$f = \{(1,2),(3,5),(4,1)\}$
$g = \{(1,3),(2,3),(5,1)\}$. Find $g \circ f$.

$$g \circ f(1) = g[f(1)] = g[2] = 3$$

$$g \circ f(3) = g[f(3)] = g[5] = 1$$

$$g \circ f(4) = g[f(4)] = g[1] = 3$$

$$g \circ f = \{(1,3),(3,1),(4,3)\}$$

⑤ If f and g are one-one functions then show that $g \circ f$ is one-one.

Since f is 1-1, $f(x) = f(y)$
$\Longrightarrow x = y$

Now $g \circ f(x) = g \circ f(y)$

$g[f(x)] = g[f(y)]$

$\cancel{g(x) =}$

$f(x) = f(y)$ since g is 1-1

$x = y$ since f is 1-1

$\therefore g \circ f$ is one-one