

## ② Inverse Trigonometric Functions.

Focus area class-1

Function	Domain	Principal value branch
$\sin^{-1}x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\cot^{-1}x$	$\mathbb{R}$	$(0, \pi)$
$\sec^{-1}x$	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \{\frac{\pi}{2}\}$
$\operatorname{cosec}^{-1}x$	$\mathbb{R} - (-1, 1)$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

### Note

The value of an inverse trigonometric function which lies in the principal branch is called principal value of the inverse trigonometric function.

① Find principal value of the following.

1,  $\sin^{-1}(\frac{1}{2})$

Let  $\sin^{-1}(\frac{1}{2}) = y$

$$\frac{1}{2} = \sin y$$

$$\sin \frac{\pi}{6} = \sin y$$

$$\therefore \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$$

$$2, \tan^{-1}(1) = \frac{\pi}{4}$$

$$3, \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$4, \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$5, \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$6, \underline{\operatorname{cosec}^{-1}(2)}$$

$$\text{Let } \operatorname{cosec}^{-1}(2) = y$$

$$2 = \operatorname{cosec} y$$

$$2 = \frac{1}{\sin y}$$

$$\sin y = \frac{1}{2}$$

$$\therefore y = \underline{\frac{\pi}{6}}$$

### Properties

$$1, \operatorname{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right), x \geq 1 \text{ or } x \leq -1$$

$$2, \operatorname{sec}^{-1}x = \cos^{-1}\left(\frac{1}{x}\right), x \geq 1, x \leq -1$$

$$3, \operatorname{cot}^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$$

Find Principal value of the following.

$$1, \operatorname{cot}^{-1}(\sqrt{3}) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

$$2, \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ = \frac{\pi}{6}$$

$$3, \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = y$$

$$-\frac{1}{2} = \sin y$$

$$-\sin \frac{\pi}{6} = \sin y$$

$$\sin\left(-\frac{\pi}{6}\right) = \sin y$$

$$\therefore y = \underline{\underline{-\frac{\pi}{6}}}$$

$$\text{or } \left. \begin{aligned} \sin^{-1}\left(-\frac{1}{2}\right) \\ &= -\sin^{-1}\left(\frac{1}{2}\right) \\ &= \underline{\underline{-\frac{\pi}{6}}} \end{aligned} \right\}$$

### Properties

$$4, \sin^{-1}(-x) = -\sin^{-1}x, x \in [-1, 1]$$

$$5, \tan^{-1}(-x) = -\tan^{-1}x, x \in \mathbb{R}$$

$$6, \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x, |x| \geq 1$$

$$7, \cos^{-1}(-x) = \pi - \cos^{-1}x, x \in [-1, 1]$$

$$8, \sec^{-1}(-x) = \pi - \sec^{-1}x, |x| \geq 1$$

$$9, \operatorname{cot}^{-1}(-x) = \pi - \operatorname{cot}^{-1}x, x \in \mathbb{R}$$

Find Principal value of the following.

$$1, \operatorname{cot}^{-1}(-\sqrt{3}) = \pi - \operatorname{cot}^{-1}(\sqrt{3}) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$2, \sec^{-1}(-\sqrt{2}) = \pi - \sec^{-1}(\sqrt{2}) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$3, \cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$