Inverse Trigonometric Functions
Focus area class-2

1, Find value of $\cos^{-1}(1/2) + 2\sin^{-1}(1/2)$

$\cos^{-1}(1/2) = \pi/3$

$\sin^{-1}(1/2) = \pi/6$

$\cos^{-1}(1/2) + 2\sin^{-1}(1/2) = \frac{\pi}{3} + 2\times\frac{\pi}{6} = \frac{2\pi}{3}$

2, Find $\sin[\pi/3 - \sin^{-1}(-1/2)]$

$\sin^{-1}(-1/2) = -\sin^{-1}(1/2) = -\pi/6$

$\sin[\pi/3 - \sin^{-1}(-1/2)] = \sin[\pi/3 - (-\pi/6)]$

$= \sin[\pi/2] = 1$

3, $\tan^{-1}[2\cos(2\sin^{-1}\frac{1}{2})]$

$= \tan^{-1}[2\cos(2\times\frac{\pi}{6})]$

$= \tan^{-1}[2\cos\pi/3]$

$= \tan^{-1}[2\times\frac{1}{2}]$

$= \tan^{-1}(1) = \pi/4$

Properties

11, $\sin^{-1}[\sin x] = x$, $x \in [-\pi/2, \pi/2]$

12, $\cos^{-1}[\cos x] = x$, $x \in [0, \pi]$

13, $\tan^{-1}[\tan x] = x$, $x \in (-\pi/2, \pi/2)$

14, $\sec^{-1}(\sec x) = x$

15, $\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$

16, $\cot^{-1}(\cot x) = x$

(17) $\sin[\sin^{-1} x] = x$, $x \in [-1, 1]$

(18) $\cos[\cos^{-1} x] = x$, $x \in [-1, 1]$

(19) $\tan[\tan^{-1} x] = x$, $x \in R$

(20) $\sec[\sec^{-1} x] = x$, $x \in R - (-1, 1)$

(21) $\cot[\cot^{-1} x] = x$, $x \in R$

(22) $\operatorname{cosec}[\operatorname{cosec}^{-1} x] = x$, $x \in R - (-1, 1)$.

Find values of the following.

1, $\sin^{-1}[\sin(\pi/6)] = \pi/6$

2, $\sin^{-1}[\sin(3\pi/5)] = 3\pi/5 \notin [-\pi/2, \pi/2]$

$\therefore \sin^{-1}\left[\sin\left(\frac{3\pi}{5}\right)\right] = \sin^{-1}\left[\sin\left(\pi - \frac{3\pi}{5}\right)\right]$

$= \sin^{-1}\left[\sin\frac{2\pi}{5}\right]$

$= \frac{2\pi}{5} \in [-\pi/2, \pi/2]$

$\sin(\pi - x) = \sin x$
$\cos(2\pi - x) = \cos x$
$\tan(x - \pi) = \tan x$

3, $\tan^{-1}\left[\tan\frac{3\pi}{4}\right] = \frac{3\pi}{4} \notin (-\pi/2, \pi/2)$

$\therefore \tan^{-1}\left[\tan 3\pi/4\right] = \tan^{-1}\left[\tan\left(\frac{3\pi}{4} - \pi\right)\right]$

$= \tan^{-1}\left[\tan(-\pi/4)\right] = -\pi/4$

4, $\cos^{-1}\left[\cos(7\pi/6)\right] = \frac{7\pi}{6} \notin [0, \pi]$

$\therefore \cos^{-1}\left[\cos\left(\frac{7\pi}{6}\right)\right] = \cos^{-1}\left[\cos\left(2\pi - \frac{7\pi}{6}\right)\right]$

$= \cos^{-1}\left[\cos\frac{5\pi}{6}\right] = \frac{5\pi}{6}$

Properties

23, $\sin^{-1}x + \cos^{-1}x = \pi/2$, $x \in [-1, 1]$

24, $\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$, $|x| \geq 1$

25, $\tan^{-1}x + \cot^{-1}x = \pi/2$, $x \in R$

① Find $\cot[\tan^{-1}a + \cot^{-1}a]$

$= \cot[\pi/2] = 0$

② Find $\sin[\sin^{-1}x + \cos^{-1}x] = \sin\pi/2 = 1$

3, Find value of x if $\sin\left[\sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}x\right]=1$

$$\sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}x=\sin^{-1}(1)$$

$$\cos^{-1}x=\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{5}\right)$$

$$x=\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{5}\right)\right]$$

$$x=\sin\left[\sin^{-1}\left(\frac{1}{5}\right)\right]$$

$$x=\frac{1}{5}$$