

MATRICES

Focus area class-1

Matrix

A matrix is a rectangular array of numbers or functions arranged into rows and columns.

If a matrix has m rows and n columns then its order is $m \times n$ and it contains mn elements.

$$\text{eg: } A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \quad O(A) = 2 \times 2$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad O(B) = 2 \times 3$$

$$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad O(C) = 3 \times 1$$

General form of a 3×3 matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

① Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ write a_{21}, a_{33}, a_{12} .

$a_{21} = 4, a_{33} = 7, a_{12} = 2$

② A matrix has 18 elements, what are the possible orders it can have?

$1 \times 18, 2 \times 9, 3 \times 6, 6 \times 3, 9 \times 2, 18 \times 1$

③ Construct a 2×2 matrix where $a_{ij} = (i+j)^2$

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$\therefore A = \begin{bmatrix} 4 & 9 \\ 9 & 16 \end{bmatrix}$

$a_{11} = 2^2 = 4$

$a_{12} = 3^2 = 9$

$a_{21} = 3^2 = 9$

$a_{22} = 4^2 = 16$

④ Construct a 2×3 matrix where $a_{ij} = 2i - j$

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$

$a_{11} = 1$

$a_{12} = 0$

$a_{13} = -1$

$a_{21} = 3$

$a_{22} = 2$

$a_{23} = 1$

Equality of matrices

Two matrices A and B are said to be equal if they have same order and each element of A is equal to corresponding element of B.

① Find value of x , y and z if

$$\begin{bmatrix} x & y \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 3 & 3 \end{bmatrix}$$

$$x = -3, y = 0, z = 2$$

② Find values of a , b , c and d if

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

$$a-b = -1 \quad \text{--- ①}$$

$$2a-b = 0 \quad \text{--- ②}$$

$$\text{②} - \text{①} \Rightarrow a = 1$$

$$\text{①} \Rightarrow 1 - b = -1 \\ b = 2$$

$$2a + c = 5$$

$$c = 3$$

$$3c + d = 13$$

$$9 + d = 13$$

$$d = \underline{\underline{4}}$$

③ Find values of x, y, z if

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

$$x+y+z = 9 \text{ ——— ①}$$

$$x+z = 5 \text{ ——— ②}$$

$$y+z = 7 \text{ ——— ③}$$

$$\text{①} - \text{②} \Rightarrow y = 4$$

$$\text{①} - \text{③} \Rightarrow x = 2$$

$$\text{①} \Rightarrow 2 + 4 + z = 9$$

$$z = \underline{\underline{3}}$$

Types of matrices

1, Column matrix \rightarrow A matrix is said to be a column matrix if it has only one column.

$$\text{eg: } \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

2, Row matrix \rightarrow A matrix is said to be a row matrix if it has only one row.

$$\text{eg: } [1 \ 2 \ 3], [1 \ -1]$$

3, zero matrix → A matrix in which all the elements are zero is called a zero matrix.

eg: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4, square matrix → A matrix in which the number of rows are equal to the number of columns, is said to be a square matrix.

eg: $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 2 & 1 \end{bmatrix}$

5, Diagonal matrix → A ~~matrix~~ ^{square} matrix is said to be a diagonal matrix if all its non-diagonal elements zero.

eg: $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

6, Identity matrix → A diagonal matrix in which main diagonal elements are 1 is called an identity matrix.

eg: $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$I_1 = \begin{bmatrix} 1 \end{bmatrix}$