MATRICES
Focus area class - 2

Operations on matrices

(1) Addition of two matrices

Two matrices A and B are conformable for addition if they have same order and sum is obtained by adding corresponding elements of A and B.

eg: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 9 & 6 \end{bmatrix}$

eg: $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -2 & 5 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 4 \\ 1 & 2 & 7 \end{bmatrix}$

(2) Difference of two matrices

Two matrices can be subtracted if they have same order and the difference is obtained by subtracting corresponding elements.

eg: $\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 6 \end{bmatrix}$

eg: $\begin{bmatrix} 1 & 2 & 5 \\ 0 & -3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 5 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 5 \\ -5 & -1 & -2 \end{bmatrix}$

(3) Scalar multiplication

Let A be any matrix and k is a scalar then kA is obtained by multiplying each element of A by k.

eg: $A\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$ then $2A=\begin{bmatrix}2 & -4\\ 6 & 8\end{bmatrix}$

$B=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\end{bmatrix}$ then $\frac{1}{2}B=\begin{bmatrix}\frac{1}{2} & 1 & \frac{3}{2}\\ 1 & 2 & 3\end{bmatrix}$

1, Find values of x, y, z and w if

$$2\begin{bmatrix}x & y\\ z & w\end{bmatrix}+3\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\ 4 & 6\end{bmatrix}$$

$$\begin{bmatrix}2x & 2y\\ 2z & 2w\end{bmatrix}+\begin{bmatrix}3 & -3\\ 0 & 6\end{bmatrix}=\begin{bmatrix}9 & 15\\ 12 & 18\end{bmatrix}$$

$$\begin{bmatrix}2x+3 & 2y-3\\ 2z & 2w+6\end{bmatrix}=\begin{bmatrix}9 & 15\\ 12 & 18\end{bmatrix}$$

$2x+3=9$ | $2y-3=15$ | $2w+6=18$

$2x=6$ | $2y=18$ | $2w=12$

$x=3$ | $y=9$ | $w=6$

$2z=12$

$\therefore z=6$ //

2, Find values of x, y, z and w if

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & x+y+6 \\ -1+z+w & 2w+3 \end{bmatrix}$$

$3x = x+4$ | $3y = x+y+6$

$2x = 4$ | $3y = 2+y+6 = 8+y$

$x = 2$ | $y = 4$

$3w = 2w+3$ | $3z = -1+z+w$

$w = 3$ | $2z = 2$

| $z = 1$

3, Find X and Y if $X+Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X-Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

$$X+Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad \text{---} \quad (1)$$

$$X-Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{---} \quad (2)$$

$(1)+(2) \Rightarrow 2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$$X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$(1)-(2) \Rightarrow 2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

$$Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

4, If $2X+Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$ and $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Find X ?

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

(4) Multiplication of matrices

Two matrices A and B are conformable for multiplication if number of columns of first matrix is equal to number of rows of second matrix. If $o(A) = m \times n$ and $o(B) = n \times p$ then $o(AB) = m * p$.

① eg: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2+6 & -1+10 \\ 6+12 & -3+20 \end{bmatrix} = \begin{bmatrix} 8 & 9 \\ 18 & 17 \end{bmatrix}$

② $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & -1 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 4 & -1 & 0 \\ 5 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2+8+5 & 3-2+2 & 1+0+1 \\ 0+12-5 & 0-3-2 & 0+0-1 \\ 4+16+5 & 6-4+2 & 2+0+1 \end{bmatrix}$

$= \begin{bmatrix} 15 & 3 & 2 \\ 7 & -5 & -1 \\ 25 & 4 & 3 \end{bmatrix}$

③ $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$

④ $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2+6+12 \end{bmatrix}$

$= \begin{bmatrix} 20 \end{bmatrix}$