MATRICES

Focus area class-3

Properties of matrix multiplication.

1, $AB \neq BA$ [Not commutative]

2, $A(BC) = (AB)C$ [Associative law]

3, $A(B+C) = AB + AC$ [Distributive law]

4, The product of 2 non-zero matrices may be a zero matrix.

① If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$

show that $AB \neq BA$

$$AB = \begin{bmatrix} 2+6 & 4+2 \\ -2+9 & -4+3 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 7 & -1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2-4 & 4+12 \\ 3-1 & 6+9 \end{bmatrix} = \begin{bmatrix} -2 & 16 \\ 2 & 15 \end{bmatrix}$$

$\therefore AB \neq BA$

② If $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -1 \\ 1 & 5 \end{bmatrix}$

show that $A(B+C) = AB + AC$

$$B+C = \begin{bmatrix} 6 & -1 \\ 6 & 6 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 6 & -1 \\ 6 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 12+24 & -2+24 \\ 18+6 & -3+6 \end{bmatrix}$$

$$= \begin{bmatrix} 36 & 22 \\ 24 & 3 \end{bmatrix} \text{ —— } (1)$$

$$AB = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8+20 & 0+4 \\ 12+5 & 0+1 \end{bmatrix} = \begin{bmatrix} 28 & 4 \\ 17 & 1 \end{bmatrix}$$

$$AC = \begin{bmatrix} 4+4 & -2+20 \\ 6+1 & -3+5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 18 \\ 7 & 2 \end{bmatrix}$$

$$AB+AC = \begin{bmatrix} 36 & 22 \\ 24 & 3 \end{bmatrix} \text{ —— } (2)$$

From (1) and (2)

$$A(B+C) = AB+AC$$

3) If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ (i) Find A^2.

(ii) Show that $A^2-5A+7I=0$

(i) $A^2=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$

(ii) $A^2-5A+7I=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$=\begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=0$

4) If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$, Find K if $A^2=KA-2I$

$A^2=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$

$=\begin{bmatrix}9-8 & -6+4\\ 12-8 & -8+4\end{bmatrix}=\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}$

Given $A^2=KA-2I$

$A^2+2I=KA$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3K & -2K \\ 4K & -2K \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3K & -2K \\ 4K & -2K \end{bmatrix}$$

$$3K = 3$$

$$\therefore K = \underline{\underline{1}}$$

5, Find value of x if

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\begin{bmatrix} 0+4+4x \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$4+4x = 0$$

$$4x = -4$$

$$\underline{\underline{x = -1}}$$

6, Find matrix B if

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a+3c & b+3d \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$c = 0$ and $d = 1$

$a+3c = 1$, $b+3d = -1$

$a = 1$ $\quad b = -1-3 = -4$

$$\therefore B = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$