

DETERMINANTS

Focus area class-1

Determinants

Every square matrix can be associated with a number or an expression is called determinant of that matrix. The determinant of a matrix A is denoted by $|A|$ or $\det A$.

Determinant of a 1×1 matrix.

$$\text{Let } A = [a] \text{ then } |A| = |a| = a$$

$$\text{eg: } A = [5] \text{ then } |A| = |5| = 5$$

$$B = [-3] \text{ then } |B| = |-3| = -3$$

Determinant of a 2×2 matrix

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then}$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\text{eg: } \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2$$

$$\text{eg: } \begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = 3 - (-8) = 3 + 8 = 11$$

$$\text{eg: } \begin{vmatrix} -2 & 1 \\ -4 & -2 \end{vmatrix} = 4 - (-4) = 4 + 4 = 8$$



Determinant of a 3x3 matrix

$$\text{Let } A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & 5 \\ 2 & 0 & 1 \end{bmatrix} \text{ then}$$

$$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 4 & 1 & 5 \\ 2 & 0 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & 5 \\ 0 & 1 \end{vmatrix} - 3 \begin{vmatrix} 4 & 5 \\ 2 & 1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 1 \\ 2 & 0 \end{vmatrix}$$

$$= 2(1-0) - 3(4-10) + (0-2)$$

$$= 2 + 18 - 2 = \underline{\underline{18}}$$

① Evaluate $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

$$\Delta = 3 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= 3(1+6) + 4(1+4) + 5(3-2)$$

$$= 21 + 20 + 5$$

$$= \underline{\underline{46}}$$

② Find $\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$

$$\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = \cos^2 \theta + \sin^2 \theta = 1$$

③ Find $\begin{vmatrix} \cos 80^\circ & -\cos 10^\circ \\ \sin 80^\circ & \sin 10^\circ \end{vmatrix}$

$$\begin{aligned} \Delta &= \cos 80^\circ \cdot \sin 10^\circ + \sin 80^\circ \cos 10^\circ \\ &= \sin [80^\circ + 10^\circ] = \sin 90^\circ = \underline{\underline{1}} \end{aligned}$$

④ Find value of x if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$

$$2 - 20 = 2x^2 - 24$$

$$2x^2 = 6$$

$$x^2 = 3$$

$$x = \underline{\underline{\pm \sqrt{3}}}$$

⑤ Find value of x if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$$10 - 12 = 5x - 6x$$

$$-2 = -x$$

$$x = \underline{\underline{2}}$$

