Determinants
Focus area Class-2

Minors

The minor of an element in a determinant is a determinant which is obtained by deleting the row and column in which element appears. The minor of an element in the i^{th} row and j^{th} column is denoted by M_{ij}.

Cofactors

The signed minor is called cofactor. The cofactor of an element in the i^{th} row and j^{th} column is denoted by C_{ij} or A_{ij} and is defined as A_{ij} or $C_{ij} = (-1)^{i+j} M_{ij}$

① Find minor and cofactors of all the elements of $\begin{vmatrix} 2 & -3 \\ 4 & 1 \end{vmatrix}$

$M_{11} = |1| = 1$ | $C_{11} = 1$

$M_{12} = |4| = 4$ | $C_{12} = -4$

$M_{21} = |-3| = -3$ | $C_{21} = 3$

$M_{22} = |2| = 2$ | $C_{22} = 2$

(2) Find cofactors of third column of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & -1 \\ 5 & 2 & 1 \end{vmatrix}$ and hence find value of the determinant.

$M_{13} = \begin{vmatrix} 4 & 0 \\ 5 & 2 \end{vmatrix} = 8$ | $A_{13} = 8$

$M_{23} = \begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} = 2 - 10$
$= -8$ | $A_{23} = 8$

$M_{33} = \begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix} = 0 - 8$
$= -8$ | $A_{33} = -8$

Now $\Delta = 3 \times 8 + -1 \times 8 + 1 \times -8$
$= 24 - 8 - 8$
$= 24 - 16 = \underline{\underline{8}}$

Note

Sum of product of elements of any row (column) with their corresponding cofactors gives the value of the determinant.

(3) Consider the determinant $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

S.T $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33} = 0$

$a_{11} = 1 \quad a_{12} = 0 \quad a_{13} = 4$

$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$ | $A_{31} = -20$

$M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13$ | $A_{32} = 13$

$M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$ | $A_{33} = 5$

$LHS = a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33}$

$= 1 \times -20 + 0 \times 13 + 4 \times 5$

$= -20 + 0 + 20$

$= \underline{\underline{0}}$

Note

If elements of a row (column) are multiplied with cofactors of any other row (column), then their sum is zero.

Singular matrix

A matrix A is said to be singular if $|A| = 0$, otherwise the matrix is said to be non-singular.

1. Find value of k if $A = \begin{bmatrix} 1 & 2 \\ k & 4 \end{bmatrix}$ is singular.

Since A is singular matrix

$$|A| = 0$$

$$\begin{vmatrix} 1 & 2 \\ k & 4 \end{vmatrix} = 0$$

$$4 - 2k = 0$$

$$2k = 4$$

$$k = \underline{\underline{2}}$$

2. Find value of k if $A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ k & 2 & 1 \end{bmatrix}$ is singular.

Since A is singular matrix, $|A| = 0$

$$\begin{vmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ k & 2 & 1 \end{vmatrix} = 0$$

$$-1 \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 0 \begin{vmatrix} 2 & 1 \\ k & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ k & 2 \end{vmatrix} = 0$$

$$-(1-2) + (4-k) = 0$$

$$1 + 4 - k = 0$$

$$k = \underline{\underline{5}}$$