Determinants
Focus area class - 3

Adjoint of a matrix

Let A be any square matrix, then its adjoint is denoted by adj A. The adjoint of a matrix is the transpose of the cofactor matrix.

① If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find adj A

$A_{11} = |4| = 4$ | $A_{12} = -|3| = -3$

$A_{21} = -|2| = -2$ | $A_{22} = |1| = 1$

Cofactor matrix of $A = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

$\therefore \text{adj } A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Note

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

② Find adj A, if $A = \begin{bmatrix} 1 & -2 \\ 5 & 4 \end{bmatrix}$

$\text{adj } A = \begin{bmatrix} 4 & 2 \\ -5 & 1 \end{bmatrix}$

(3) Find adj A, if $A = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

$$A_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10+1 = 11$$

$$A_{12} = -\begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = -6$$

$$A_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$$

$$A_{21} = -\begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = -(-4) = 4$$

$$A_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2$$

$$A_{23} = -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$$

$$A_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = -20$$

$$A_{32} = -\begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -(-1-12) = 13$$

$$A_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5$$

$$\therefore \operatorname{adj} A = \begin{bmatrix} 11 & 4 & -20 \\ -6 & 2 & 13 \\ 3 & -1 & 5 \end{bmatrix}$$

Short cut method

(4) Find adj A, if $A = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

$$\begin{array}{c|cccc} 1 & 0 & 4 & 1 & 0 \\ \hline 3 & 5 & -1 & 3 & 5 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 0 & 4 & 1 & 0 \\ 3 & 5 & -1 & 3 & 5 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 10+1 & 4 & -20 \\ -6 & 2 & 12+1 \\ 3 & -1 & 5 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 11 & 4 & -20 \\ -6 & 2 & 13 \\ 3 & -1 & 5 \end{bmatrix}$$

H.W

(5) Find adj A, if $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$