Determinants

Focus area Class-4

Properties of adjoint of a matrix

① $A(\operatorname{adj} A) = (\operatorname{adj} A)\cdot A = |A| I$

② $|\operatorname{adj} A| = |A|^{n-1}$, where $O(A) = n \times n$

③ $\operatorname{adj}(\operatorname{adj} A) = |A|^{n-2} \cdot A$, where $O(A) = n \times n$

④ $\operatorname{adj}(AB) = \operatorname{adj} B \cdot \operatorname{adj} A$

⑤ $\operatorname{adj}(A^T) = (\operatorname{adj} A)^T$

Inverse of a matrix

Let A be any square matrix then its inverse is denoted by A^{-1}.

$$\therefore A^{-1} = \frac{\operatorname{adj} A}{|A|}, \quad |A| \neq 0$$

① Consider the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

Prove that (i) $A \cdot \operatorname{adj} A = |A| \cdot I$

(ii) Find A^{-1}.

(i) $\operatorname{adj} A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

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$$A\cdot(\text{adj}\,A) = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8-3 & -6+6 \\ 4-4 & -3+8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \text{ — (1)}$$

$$|A| = 8-3 = 5$$

$$|A|I = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \text{ — (2)}$$

From (1) and (2):

$$A\cdot \text{adj}\,A = |A|\,I$$

(ii) $$A^{-1} = \frac{\text{adj}\,A}{|A|} = \frac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

(2) Find $A\cdot\text{adj}\,A$, if $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$

$$|A| = 12-10 = 2$$

$$A\cdot\text{adj}\,A = |A|\,I = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

(3) Let $A = \begin{bmatrix} 5 & -2 \\ 4 & -1 \end{bmatrix}$, find $|\text{adj}\,A|$

$$|A| = -5+8 = 3$$

$$|\text{adj}\,A| = |A|^{2-1} = |A| = 3$$

(4) Find inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$

$$|A| = 1\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= (1+6) - (0-3) + (0-1)$$

$$= 7+3-1 = 9$$

$$A_{11} = \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = 1+6 = 7$$

$$A_{12} = -\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = -(0-3) = 3$$

$$A_{13} = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = 0-1 = -1$$

$$A_{21} = -\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -(1+2) = -3$$

$$A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -(-2-1) = 3$$

$$A_{31} = \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 3-1 = 2$$

$$A_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = -3$$

$$A_{33} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

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$$\therefore \text{adj} A = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

(5) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$ show that

$$A \cdot \text{adj} A = |A| I.$$

$$RHS = |A| I = 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \text{ --- (1)}$$

$$LHS = A \cdot \text{adj} A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \text{ --- (2)}$$

From (1) and (2)

$$A \cdot \text{adj} A = |A| I$$

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