Determinants

Focus area Class-5

Properties of inverse of a matrix

1. $AA^{-1} = A^{-1}.A = I$
2. $(AB)^{-1} = B^{-1}.A^{-1}$
3. $(A^{T})^{-1} = (A^{-1})^{T}$
4. $|A^{-1}| = \dfrac{1}{|A|}$
5. $(A^{-1})^{-1} = A$

1, If $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

show that $(AB)^{-1} = B^{-1}.A^{-1}$

$$AB = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$$|AB| = 67\times61 - 47\times87$$
$$= 4087 - 4089 = -2$$

$$(AB)^{-1} = \frac{adj(AB)}{|AB|} = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$\to$ (1)

$$|B| = 54 - 56 = -2$$

$$B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$|A| = 15 - 14 = 1$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{-1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{-1}{2}\begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$$

$$= \frac{-1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \text{——} (2)$$

From (1) and (2) $\underline{(AB)^{-1} = B^{-1} \cdot A^{-1}}$

2, If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $\underline{A^2 - 5A + 7I = 0}$ and hence find A^{-1} and A^3.

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$LHS = A^2 - 5A + 7I$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Now $A^2 - 5A + 7I = 0$

Premultiply each term by A^{-1}

$$A^{-1}A^2 - 5A^{-1}A + 7A^{-1}I = 0$$

$$A - 5I + 7A^{-1} = 0$$

$$7A^{-1} = 5I - A$$

$$7A^{-1} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Now $A^2 - 5A + 7I = 0$

Xig by A, $A^3 - 5A^2 + 7IA = 0$

$$A^3 = 5A^2 - 7A$$

$$= \begin{bmatrix} 40 & 25 \\ -25 & 15 \end{bmatrix} - \begin{bmatrix} 21 & 7 \\ -7 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 18 \\ -18 & 1 \end{bmatrix}$$