Determinants

Focus area class-6

Solution of system of linear equations.

Consider the system of linear equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

The given system can be written in the matrix form $AX = B$

where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$,

$B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Case (i)

If $|A| \neq 0$, then the system have only one solution.

$$X = A^{-1}B$$

Case (ii)

If $|A| = 0$ and $(\text{adj}A)\cdot B \neq 0$, then the system has no solution.

Case (iii)

If $|A| = 0$ and $(\text{adj} A) \cdot B = 0$, then the system has either many solutions or no solution.

Note

A system of equation is said to be consistent if it has one or more solutions. otherwise the system is said to be inconsistent.

(1) Solve using matrix method
$x + 2y = 2$ and $2x + 3y = 3$

The given equation can be written in the matrix form $AX = B$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$|A| = 3 - 4 = -1 \neq 0$

$\therefore$ The system have only one solution.

Hence the system is consistent.

Now $\text{adj} A = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$

$$A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{-1} \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

Now $X = A^{-1} \cdot B = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$= \begin{bmatrix} -6+6 \\ 4-3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\therefore x = 0, \ y = 1$

(2) Check whether the system is consistent if consistent solve it.

$x + 3y = 5$ and $2x + 6y = 8$

The given equations can be written in the matrix form $AX = B$

$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, \ X = \begin{bmatrix} x \\ y \end{bmatrix}, \ B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$|A| = 6 - 6 = 0$

Now $\operatorname{adj} A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

$(\operatorname{adj} A) B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$$= \begin{bmatrix} 30-24 \\ -10+8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$$

$\therefore$ The system has no solution.
Hence the system is inconsistent.

3, Solve using matrix method

$2x - y = 5$ and $x + y = 4$

The given equations can be written in the matrix form $AX = B$.

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$|A| = 2 + 1 = 3 \neq 0$$

$\therefore$ The system has only one solution.

Hence the system is consistent.

$$A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$$

Now $X = A^{-1} B$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\therefore x = 3, \; y = 1$