Determinants
Focus area Class-7

(1) Solve using matrix method

$2x-3y+5z=11$, $3x+2y-4z=-5$

and $x+y-2z=-3$.

The given equations can be written in the matrix form $AX=B$.

$$A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\quad B=\begin{bmatrix}11\\ -5\\ -3\end{bmatrix}$$

$$|A|=2\begin{vmatrix}2 & -4\\ 1 & -2\end{vmatrix}+3\begin{vmatrix}3 & -4\\ 1 & -2\end{vmatrix}+5\begin{vmatrix}3 & 2\\ 1 & 1\end{vmatrix}$$

$$=2(-4+4)+3(-6+4)+5(3-2)$$

$$=0-6+5=-1\neq 0$$

∴ The system has only one solution

Hence the system is consistent.

$$A_{11}=\begin{vmatrix}2 & -4\\ 1 & -2\end{vmatrix}=-4+4=0$$

$$A_{12}=-\begin{vmatrix}3 & -4\\ 1 & -2\end{vmatrix}=-(-6+4)=2$$

$$A_{13}=\begin{vmatrix}3 & 2\\ 1 & 1\end{vmatrix}=3-2=1$$

$$A_{21}=-\begin{vmatrix}-3 & 5\\ 1 & -2\end{vmatrix}=-(6-5)=-1$$

$$A_{22} = \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} = -4-5 = -9$$

$$A_{23} = -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2+3) = -5$$

$$A_{31} = \begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} = 12-10 = 2$$

$$A_{32} = -\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} = -(-8-15) = 23$$

$$A_{33} = \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 4+9 = 13$$

$$\text{adj}\,A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}\,A}{|A|} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore x = 1,\ y = 2 \text{ and } z = 3$$

(2) Check whether the system is consistent or not if consistent solve it.

$3x - y - 2z = 2,\ 2y - z = -1,\ 3x - 5y = 3$

The given equations can be written in the matrix form $AX = B$

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$|A| = 3(0-5) + 1(0+3) - 2(0-6)$$
$$= -15 + 3 + 12 = 0$$

$$\begin{array}{c|cccc} 3 & -1 & -2 & 3 & -1 \\ \hline 0 & 2 & -1 & 0 & 2 \\ 3 & -5 & 0 & 3 & -5 \\ 3 & -1 & -2 & 3 & -1 \\ 0 & 2 & -1 & 0 & 2 \end{array}$$

$$\operatorname{adj} A = \begin{bmatrix} 0-5 & 10-0 & 1+4 \\ -3 & 6 & 3 \\ -6 & -3+15 & 6-0 \end{bmatrix}$$

$$\operatorname{adj} A = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(\operatorname{adj} A)\cdot B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10-10+15 \\ -6-6+9 \\ -12-12+18 \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$$

$\therefore$ The system has no solution.
Hence the system is inconsistent.

(3) The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Find the numbers using matrix method.

Let x, y, z be the numbers.

$$x+y+z=6$$

$$y+3z=11$$

$$x-2y+z=0$$

The given equations can be written in the matrix form $AX=B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$|A| = 1(1+6) - (0-3) + (0-1) = 9 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

Now $X = A^{-1} \cdot B$

$$= \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 42 - 33 + 0 \\ 18 + 0 + 0 \\ -6 + 33 + 0 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1$, $y = 2$ and $z = 3$