SOLUTION OF SYSTEM OF LINEAR EQUATIONS

Consider the system of linear eqns

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

The above system of eqns can be expressed in the matrix form $AX = B$, where

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Case (i)

If $|A| \neq 0$, then the system have only one solution

$$\boxed{X = A^{-1} \cdot B}$$

Case (ii)

If $|A| = 0$ & $(\text{adj}A) \cdot B \neq 0$ then the system have no solution

Case (iii)

If $|A| = 0$ & $(\text{adj}A) \cdot B = 0$ then the system has either infinite solution / no solution.

Note:

A system of eqn is said to be consistant if it have solution otherwise the system is said to be inconsistant.

1. Check the consistance of $x+2y=2$ & $2x+3y=3$. If consistant then solve it.

The given eqns can be written in the matrix form $AX=B$, where

$$A=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix} \quad X=\begin{bmatrix}x\\ y\end{bmatrix} \quad B=\begin{bmatrix}2\\ 3\end{bmatrix}$$

$|A| = 3-4 = -1 \neq 0$

∴ The system have only one solution

Hence the system is consistent.

$$\text{adj } A=\begin{bmatrix}3 & -2\\ -2 & 1\end{bmatrix}$$

$$A^{-1}=\frac{\text{adj } A}{|A|}$$

$$=\frac{1}{-1}\begin{bmatrix}3 & -2\\ -2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-3 & 2\\ 2 & -1\end{bmatrix}$$

$$X=A^{-1}.B$$

$$= \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -6+6 \\ 4-3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\therefore x = 0 \quad y = 1$

2. Solve using matrix method $2x + 5y = 1$

$3x + 2y = 7$.

The given eqns can be written on the matrix form $AX = B$ where

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$|A| = 4 - 15 = -11 \neq 0$

$\therefore$ The system have only one solution Hence the system is consistend.

$$\text{adj } A = \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{1}{-11} \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

$$= \frac{1}{-11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$= \frac{1}{-11}\begin{bmatrix} -33 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$\therefore x = 3 \quad y = -1$

3 Check the consistence of $x+3y=5$, $2x+6y=8$. If consistent then solve it.

The given eqns can be written in the matrix form $AX = B$ where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$|A| = 6-6 = 0$

$$\text{adj } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

$$(\text{adj } A) \cdot B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$\neq 0$

$\therefore$ No solution

Hence the system is inconsistent.