# Class-15

## Rolle's theorem

If $f$ is a real function defined $[a,b]$ such that

(i) $f$ is continuous on $[a,b]$

(ii) $f$ is differentiable on $(a,b)$

(iii) $f(a) = f(b)$

then there exist atleast one point $c \in (a,b)$ such that $f'(c) = 0$

① verify Rolle's theorem for $f(x) = x^2+2$, $a=-2$, $b=2$.

clearly $f(x)$ is continuous on $[-2,2]$ since $f(x)$ is a polynomial function.

Now $f'(x) = 2x$

$\therefore f(x)$ is differentiable on $(-2,2)$

$f(-2) = 6$ and $f(2) = 6$

$\therefore f(-2) = f(2)$

Now $f'(c) = 0$

$$2c = 0$$
$$c = 0 \in (-2, 2)$$
Hence theorem is verified.

(2) Verify Rolle's theorem for
$f(x) = x^2 + 2x - 8$ in $[-4, 2]$

Since $f(x)$ is a Polynomial function
$f(x)$ is continuous on $[-4, 2]$.

Now $f'(x) = 2x + 2$

$\therefore f(x)$ is differentiable on $(-4, 2)$

$$f(-4) = 16 - 8 - 8 = 0$$
$$f(2) = 4 + 4 - 8 = 0$$
$$\therefore f(-4) = f(2)$$

Now $f'(c) = 0$
$$2c + 2 = 0$$
$$2c = -2$$
$$c = -1 \in (-4, 2)$$

Hence theorem is verified

3, Examine the applicability of

$f(x) = x^2 - 1$ in $[1, 2]$

Since $f(x)$ is a Polynomial function

$f(x)$ is continuous in $[1,2]$.

Now $f'(x) = 2x$

$\therefore f(x)$ is differentiable on $(1, 2)$

$f(1) = 0$ and $f(2) = 3$

$f(1) \neq f(2)$

Hence Rolle's theorem is not applicable.

4, Examine the applicability of

$f(x) = [x]$ in $[5, 9]$

$f(x)$ is not continuous on $[5, 9]$

$\therefore$ Rolle's theorem is not applicable.