Class - 16

Lagrange's mean Value Theorem

Mean value Theorem

Let f be a real function on $[a,b]$ such that

(i) f is continuous in $[a,b]$

(ii) f is differentiable in (a,b)

then there exist atleast one Point $c \in (a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

① verify mean value theorem for $f(x) = x^2$ in $[2,4]$

since $f(x)$ is a Polynomial function it is continuous on $[2,4]$.

$f'(x) = 2x$

$\therefore f(x)$ is differentiable on $(2,4)$

$f(a) = f(2) = 4$

$$f(b) = f(4) = 16$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$2c = \frac{16 - 4}{4 - 2} = \frac{12}{2} = 6$$

$$2c = 6$$

$$c = 3 \in (2, 4)$$

Hence the theorem is verified.

(2) Verify M.V.T for $f(x) = x^2 - 4x - 3$ in $[1, 4]$.

Since $f(x)$ is a Polynomial function it is continuous on $[1, 4]$

$$f'(x) = 2x - 4$$

$\therefore$ $f(x)$ is differentiable on $(1, 4)$

$$f(a) = f(1) = -6$$

$$f(b) = f(4) = 16 - 16 - 3 = -3$$

Now $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$$2c-4 = \frac{-3+6}{3}$$

$$2c-4 = 1$$

$$c = 5/2 \in (1, 4)$$

Hence theorem is verified.

③ verify M.V.T $f(x) = x^3 - 5x^2 - 3x$ in $[1,3]$.

Since $f(x)$ is a Polynomial function it is continuous on $[1,3]$.

$$f'(x) = 3x^2 - 10x - 3$$

$\therefore f(x)$ is differentiable in $(1,3)$.

$$f(a) = f(1) = 1-5-3 = -7$$

$$f(b) = f(3) = -27$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$3c^2 - 10c - 3 = \frac{-27+7}{2} = -10$$

$$3c^2 - 10c + 7 = 0$$

$$c = \frac{10 \pm \sqrt{100 - 4 \times 3 \times 7}}{6}$$

$$c = \frac{10 \pm 4}{6}$$

$$c = \frac{7}{3} \text{ or } 1$$

$$c = \frac{7}{3} \in (1,3)$$

Hence the theorem is verified.

(4) verify M.V.T for $f(x) = e^x$ in $[0,1]$

Clearly $f(x)$ is continuous in $[0,1]$

$f'(x) = e^x$

$\therefore f(x)$ is differentiable on $(0,1)$

$f(a) = f(0) = e^0 = 1$

$f(b) = f(1) = e$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$e^c = \frac{e-1}{1}$$

$e^c = e - 1$

$\log(e^c) = \log(e-1)$

$c \cdot \log e = \log(e-1)$

$c = \log(e-1) \in (0,1)$

Hence theorem is verified.

$\log e = 1$

$1 < e-1 < e$

$\log 1 < \log(e-1) < \log e$

$0 < \log(e-1) < 1$

(5) Verify M.V.T for $f(x) = \log x$ in $[1, e]$

Clearly $f(x)$ is continuous in $[1, e]$

$$f'(x) = \frac{1}{x}$$

$\therefore f(x)$ is differentiable on $(1, e)$

$$f(a) = f(1) = \log 1 = 0$$

$$f(b) = f(e) = \log e = 1$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\frac{1}{c} = \frac{1}{e-1}$$

$$c = e - 1 \in (1, e)$$

Hence the theorem is verified.

(6) Examine the applicability of

(i) $f(x) = [x]$ in $[5, 9]$ (ii) $f(x) = |x|$ in $[-1, 1]$

(i) $f(x)$ is not continuous in $[5, 9]$

Hence M.V.T is not applicable.

(ii) $f(x)$ is continuous in $[-1, 1]$

$|x|$ is not differentiable at $x = 0$

$\therefore f(x)$ is not differentiable in $(-1, 1)$

Hence M.V.T is not applicable.