Class-1

Application of Derivatives.

Rate of change of quantities

(1) Find rate of change of area of a circle with respect to its radius r when $r = 3\,cm$

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$= 2\pi \times 3$

$= 6\pi\ cm^2/cm$

$A \rightarrow$ area

$r \rightarrow$ radius

$r = 3\,cm$

(2) The radius of a circle is increasing at the rate of 3cm/s. Find rate of change of area of the circle is increasing when radius is 10 cm.

$A = \pi r^2$

diff: w.r.t t

$\frac{dA}{dt} = \pi \times 2r \frac{dr}{dt}$

$= \pi \times 2 \times 10 \times 3$

$= 60\pi\ cm^2/s$

$A \rightarrow$ area

$r \rightarrow$ radi

$r = 10cm$

$\frac{dr}{dt} = 3\,cm/$

(3) A stone is dropped into a quiet lake and waves moves in circles at the speed of 5cm/s, when radius of the circular wave is 8cm, how fast is the enclosed area increasing?

$A = \pi r^2$

A → area

r → radius

$r = 8\text{cm}$

$\frac{dr}{dt} = 5\text{cm/s}$

diff: w.r.t t

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$= 2\pi \times 8 \times 5$

$= \underline{80\pi \text{ cm}^3/\text{s}}$

(4) The radius of a circle is increasing at the rate of .7cm/s. what is the rate of increase of its circumference?

$C = 2\pi r$

C → circumference

$\frac{dr}{dt} = .7\text{ cm/s}$

diff: w.r.t t

$\frac{dC}{dt} = 2\pi \times \frac{dr}{dt}$

$= 2\pi \times .7$

$= \underline{1.4\pi \text{ cm/s}}$

(5) A balloon, which always remains spherical has a variable radius. Find rate of change of its volume when radius is 10cm

$V = \frac{4}{3}\pi r^3$

$V \rightarrow$ volume
$r \rightarrow$ radius

Diff: w.r.t r

$r = 10cm$

$$\frac{dV}{dr} = \frac{4}{3}\pi \times 3r^2$$

$$= 4\pi \times 100 = 400\pi \, cm^3/cm$$

(6) The radius of a air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing when $r = 1cm$.

$V = \frac{4}{3}\pi r^3$

$V \rightarrow$ volume
$r \rightarrow$ radius

Diff: w.r.t t

$\frac{dr}{dt} = \frac{1}{2}$ cm/s

$r = 1cm$

$$\frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt}$$

$$= 4\pi \times 1 \times \frac{1}{2}$$

$$= 2\pi \, cm^3/s$$