Class - 2

Application of Derivatives

Rate of change of quantities (contd)

(8) An edge of a variable cube is increasing at the rate of 3cm/s. How fast is the volume of the cube is increasing when edge is 10cm long.

$x \rightarrow$ edge of the cube

$V \rightarrow$ volume

$x = 10cm$

$\frac{dx}{dt} = 3 cm/s$

$V = x^3$

diff: w.r.t $\cancel{x}$ t

$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$

$= 3 \times 100 \times 3$

$= 900 \, cm^3/s$

(9) The volume of the cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when length of an edge is 12 cm.

$V \rightarrow$ volume

$A \rightarrow$ surface area

$x \rightarrow$ edge of the cube

$x = 12 cm$

$\frac{dV}{dt} = 8 cm^3/s$

$A = 6x^2$

diff: w.r.t t

$\frac{dA}{dt} = 6 \times 2x \cdot \frac{dx}{dt}$ — (1)

we have $v = x^3$

$$\frac{dv}{dt} = 3x^2 \frac{dx}{dt}$$

$$8 = 3 \times 144 \times \frac{dx}{dt}$$

$$\frac{8}{3 \times 144} = \frac{dx}{dt}$$

$$(1) \Rightarrow \frac{dA}{dt} = 6 \times 2x \times \frac{dx}{dt}$$

$$= 6 \times 2 \times 12 \times \frac{8}{3 \times 144}$$

$$= \underline{\underline{\frac{8}{3} \text{ cm}^2/\text{s}}}$$

(10) The length x of a rectangle is decreasing at the rate of 5 cm/min and width y is increasing at the rate of 4 cm/min. when $x = 8$ cm, $y = 6$ cm find rate of change of its (i) area (ii) Perimeter.

$$\frac{dx}{dt} = -5 \text{ cm/min}, \quad \frac{dy}{dt} = 4 \text{ cm/min}$$

$x = 8$ cm

$y = 6$ cm

(i) $A = xy$

diff: w.r.t t

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$= 8 \times 4 + 6 \times -5$$

$$= \underline{\underline{2\ cm^2/min}}$$

(ii) $P = 2x + 2y$

diff: w.r.t t

$$\frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$$

$$= 2 \times -5 + 2 \times 4$$

$$= \underline{\underline{-2\ cm/min}}$$

(11) A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is the height on the wall decreasing when foot of the ladder is 4 m away from the wall?

$x \rightarrow$ distance of the bottom of the ladder from the wall

$y \rightarrow$ height of the top of the ladder from the ground.

$\frac{dx}{dt} = 2cm/s$, $x = 4$, $\frac{dy}{dt} = ?$

we have $x^2 + y^2 = 25$

diff: w.r.t t

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$y \frac{dy}{dt} = -x \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-x}{y} \frac{dx}{dt}$$

$$= \frac{-400}{300} \times 2$$

$$= -8/3 \text{ cm/s}$$

$y^2 = 25-16$

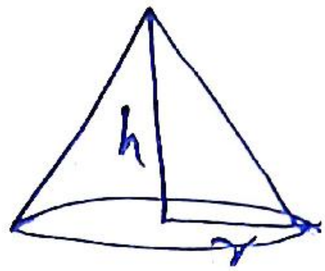
$y = 3m$

$\therefore y = 300cm$

(12) Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one sixth of the base radius. How fast is the height of the sand cone increasing when height is 4cm?

$V \to$ volume, $h =$ height, $r \Rightarrow$ base radius

Given $h = \frac{1}{6} r$



$6h = r$

$\frac{dV}{dt} = 12\ cm^3/s$, $h = 4\ cm$

Now $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 36 h^3 = 12\pi h^3$

$\frac{dV}{dt} = 12\pi \times 3h^2 \frac{dh}{dt}$

$12 = 12\pi \times 3 \times 16 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{48\pi}$ cm/s

(13) The total revenue recieved from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find marginal revenue when $x = 7$

$\frac{dR}{dx} = 13 \times 2x + 26 = 26x + 26$

marginal revenue $= \left(\frac{dR}{dx}\right)_{x=7}$

$= 26 \times 7 + 26 = 208$

(14) The total cost $C(x)$ in rupees associated with the production of x units of an item is given by

$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$

Find marginal cost when $x = 17$

$$\frac{dc}{dx} = 0.007 \times 3x^2 - 0.003 \times 2x + 15$$

$$\text{marginal cost} = \left(\frac{dc}{dx}\right)_{x=17}$$

$$= 0.007 \times 3 \times 289 - 0.003 \times 2 \times 17 + 15$$

$$= \underline{20.967}$$