Class – 8

# Increasing and decreasing functions

Let $f$ be continuous on $[a,b]$ and differentiable on $(a,b)$ then

(i) $f$ is strictly increasing in $(a,b)$ if $f'(x) > 0$ for each $x \in (a,b)$

(ii) $f$ is strictly decreasing in $(a,b)$ if $f'(x) < 0$ for each $x \in (a,b)$

(iii) $f$ is increasing in $(a,b)$ if $f'(x) \geq 0$ for each $x \in (a,b)$

(iv) $f$ is decreasing in $(a,b)$ if $f'(x) \leq 0$ for each $x \in (a,b)$

(v) $f$ is a constant function in $(a,b)$ iff $f'(x) = 0$, for each $x \in (a,b)$

(1) Show that $f(x) = 3x + 17$ is strictly increasing on $R$.



$f'(x) = 3 > 0$

$\therefore f(x)$ is strictly increasing on $R$.

(5) Show that $f(x) = x^3 - 3x^2 + 4x$, $x \in R$ is strictly increasing on R.

$$f'(x) = 3x^2 - 6x + 4$$

$$= 3(x^2 - 2x) + 4$$

$$= 3[x^2 - 2x + 1 - 1] + 4$$

$$= 3[(x-1)^2 - 1] + 4$$

$$= 3(x-1)^2 - 3 + 4$$

$$= 3(x-1)^2 + 1 > 0$$



$\therefore f(x)$ is strictly increasing on R.

(6) Show that $f(x) = \sin x$ is (i) strictly increasing in $(0, \pi/2)$ (ii) strictly decreasing in $(\pi/2, \pi)$ (iii) neither increasing nor decreasing.

$$f'(x) = \cos x$$

(i) In $(0, \pi/2)$, $\cos x$ is Positive

$\therefore f(x)$ is strictly $\uparrow$ in $(0, \pi/2)$

(ii) In $(\pi/2, \pi)$, $\cos x$ is Negative

$\therefore$ $f(x)$ is strictly $\downarrow$ in $(\pi/2, \pi)$

(iii) clearly $f(x)$ is neither increasing nor decreasing.

(7) Show that $f(x) = \log(\cos x)$ is

(i) strictly increasing in $(\pi/2, \pi)$

(ii) strictly decreasing in $(0, \pi/2)$.

$$f'(x) = \frac{1}{\cos x} \times -\sin x = -\tan x$$

(i) In $(\pi/2, \pi)$, $\tan x$ is negative

$\therefore f'(x) > 0$

$f(x)$ is strictly increasing in $(\pi/2, \pi)$

(ii) In $(0, \pi/2)$, $\tan x$ is positive

$\therefore f'(x) < 0$

$\therefore f(x)$ is strictly decreasing in $(0, \pi/2)$