Class - 9

Increasing and Decreasing Functions

① Find the intervals in which the function $f(x) = x^2 - 4x + 6$ is (i) strictly increasing (ii) strictly decreasing.

$f'(x) = 2x - 4$

$f'(x) = 0 \implies 2x - 4 = 0$

$x = 2$

The Point $x = 2$ divide the real line into two disjoint intervals $(-\infty, 2)$ and $(2, \infty)$.

In $(-\infty, 2)$, $f'(x) = 2x - 4$

$x = 0 \implies f'(0) = -4 < 0$

$\therefore f(x)$ is strictly decreasing in $(-\infty, 2)$

In $(2, \infty)$, $f'(x) = 2x - 4$

$f'(3) = 6 - 4 = 2 > 0$

$f'(x) > 0$

$\therefore f(x)$ is strictly increasing in $(2, \infty)$

(2) Find the intervals in which $f(x) = 2x^2 - 3x$ is strictly ↑ or strictly ↓.

$f(x) = 2x^2 - 3x$

$f'(x) = 4x - 3$

$f'(x) = 0 \implies 4x - 3 = 0$

$x = 3/4$

The Point $x = 3/4$ divide the real line into 2 disjoint intervals $(-\infty, 3/4)$ and $(3/4, \infty)$.

In $(-\infty, 3/4)$

$f'(x) = 4x - 3$

$f'(0) = -3$

$\therefore f'(x) < 0$

$\therefore f(x)$ is strictly decreasing in $(-\infty, 3/4)$

In $(3/4, \infty)$

$f'(x) = 4x - 3$

$f'(2) = 8 - 3 = 5 > 0$

$\therefore f'(x) > 0$

$\therefore f(x)$ is strictly increasing in $(3/4, \infty)$

(3) Find the intervals in which $f(x) = 2x^3 - 3x^2 - 36x + 7$ is (i) strictly increasing (ii) strictly decreasing.

$f'(x) = 6x^2 - 6x - 36$

$f'(x) = 0 \implies 6x^2 - 6x - 36 = 0$

$6(x^2 - x - 6) = 0$

$6(x-3)(x+2) = 0$

$x = 3$ or $x = -2$

The Points $x = -2$ and $x = 3$ divide the real line into 3 disjoint intervals $(-\infty, -2)$ $(-2, 3)$ and $(3, \infty)$.

Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, -2)$	$(-)(-) = +ve$	strictly $\uparrow$
$(-2, 3)$	$(-)(+) = -ve$	strictly $\downarrow$
$(3, \infty)$	$(+)(+) = +ve$	strictly $\uparrow$

(4) Find the intervals in which $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing or strictly decreasing.

$f(x) = -2x^3 - 9x^2 - 12x + 1$

$f'(x) = -6x^2 - 18x - 12$

$= -6\left[x^2 + 3x + 2\right]$

$= -6(x+1)(x+2)$

Now $f'(x) = 0 \Rightarrow 6(x+1)(x+2) = 0$

$\Rightarrow x = -1, x = -2$

The Points $x = -1$ and -2 divide the real line into 3 disjoint intervals $(-\infty, -2)$, $(-2, -1)$ and $(-1, \infty)$

Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, -2)$	$(-)(-)(-) = -ve$	strictly $\downarrow$
$(-2, -1)$	$(-)(-)(+) = +ve$	strictly $\uparrow$
$(-1, \infty)$	$(-)(+)(+) = -ve$	strictly $\downarrow$

5. Find the intervals in which $f(x) = \sin 3x, x \in [0, \pi/2]$ is (i) strictly $\uparrow$ (ii) strictly $\downarrow$.

$f'(x) = 3\cos 3x$

Now $f'(x) = 0 \Rightarrow \cos 3x = 0$

$\Rightarrow 3x = (2n+1)\pi/2$

$\Rightarrow x = (2n+1)\pi/6$

$\Rightarrow x = \pi/6$ or $\pi/2$

The Points $x = \pi/6, \pi/2$ divide the interval $[0, \pi/2]$ into 2 intervals $[0, \pi/6)$ and $(\pi/6, \frac{\pi}{2}]$.

In $[0, \pi/6)$

$f'(x) = 3\cos 3x > 0$

$\therefore f(x)$ is increasing in $[0, \pi/6)$

In $(\pi/6, \pi/2]$

$f'(x) = 3\cos 3x < 0$

$\therefore f(x)$ is decreasing in $(\pi/6, \pi/2]$