Integrals class -2

Type-1 : Direct Integration.

Integrate the following functions

1, $\int x^5 \, dx = \frac{x^6}{6} + c$

2, $\int 3x^2 \, dx = 3 \int x^2 \, dx = 3 \cdot \frac{x^3}{3} = x^3 + c$

3, $\int (x^{5/2} + 3)\, dx = \int x^{5/2} \, dx + \int 3 \, dx$

$= \frac{x^{7/2}}{7/2} + 3x + c$

4, $\int (2x - 3\cos x + e^x)\, dx$

$= 2 \cdot \frac{x^2}{2} - 3\sin x + e^x + c$

5, $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$= 2 \cdot \frac{x^3}{3} + 3\cos x + 5 \cdot \frac{x^{3/2}}{3/2} + c$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x^{3/2}}{3} + c$

6) $\int (2x^2 + e^x)\, dx = \frac{2 \cdot x^3}{3} + e^x + c$

7, $\int(ax^2+bx+c)\,dx$

$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$$

8, $\int x^2(1-x^2)\,dx = \int(x^2 - x^4)\,dx$

$$= \frac{x^3}{3} - \frac{x^5}{5} + c$$

9, $\int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx = \int\left(x - 2 + \frac{1}{x}\right) dx$

$$= \frac{x^2}{2} - 2x + \log|x| + c$$

10, $\int x^2\left(1 - \frac{1}{x^2}\right) dx = \int(x^2 - 1)\,dx$

$$= \frac{x^3}{3} - x + c$$

11, $\int\sqrt{x}\,(1-x)\,dx = \int(\sqrt{x} - x\sqrt{x})\,dx$

$$= \int(\sqrt{x} - x^{3/2})\,dx$$

$$= \frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + c$$

$$= \frac{2x^{3/2}}{3} - \frac{2x^{5/2}}{5} + c$$

12, $\int\left(\frac{x^3+5x^2-4}{x^2}\right) dx = \int\left(x + 5 - \frac{4}{x^2}\right) dx$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + c$$

13, $\displaystyle\int \frac{x^3 - x^2 + x - 1}{x-1}\,dx$

$\displaystyle = \int \frac{x^2(x-1) + (x-1)}{x-1}\,dx$

$\displaystyle = \int x^2\,dx + \int 1\,dx = \frac{x^3}{3} + x + c$

14, $\displaystyle\int \frac{2 - 3\sin x}{\cos^2 x}\,dx$

$\displaystyle = \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right) dx$

$\displaystyle = \int (2\sec^2 x - 3\sec x \tan x)\,dx$

$= 2\tan x - 3\sec x + c$

15, $\displaystyle\int \sec x(\sec x + \tan x)\,dx$

$\displaystyle = \int (\sec^2 x + \sec x \tan x)\,dx$

$= \tan x + \sec x + c$

16, $\displaystyle\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx = \frac{x^{3/2}}{3/2} + 2\sqrt{x} + c$

17, $\displaystyle\int (2\sin x - e^x)\,dx = -2\cos x - e^x + c$

(18) $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. then find $f(x)$.

$$\int \frac{d}{dx} f(x) = \int \left(4x^3 - \frac{3}{x^4}\right) dx$$

$$f(x) = \left[4 \cdot \frac{x^4}{4} - 3\,\frac{x^{-3}}{-3}\right] + c$$

$$f(x) = x^4 + \frac{1}{x^3} + c \quad \text{—— (1)}$$

Given $f(2) = 0$

$$2^4 + \frac{1}{2^3} + c = 8$$

$$c = -16 - \frac{1}{8} = \frac{-129}{8}$$

$$(1) \Rightarrow f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$