Integrals class-4

Type-3

$$\boxed{\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c}$$

① Integrate the following functions.

1, $\int \frac{2x}{1+x^2}\,dx = \log|1+x^2| + c$

2, $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \log|e^x + e^{-x}| + c$

3, $\int \frac{\cos x}{1+\sin x}\,dx = \log|1+\sin x| + c$

4, $\int \frac{\sin x}{1+\cos x}\,dx = -\int \frac{-\sin x}{1+\cos x}\,dx$

$= -\log|1+\cos x| + c$

5, $\int \frac{x}{9-4x^2}\,dx = \frac{1}{-8}\int \frac{-8x}{9-4x^2}\,dx$

$= \frac{-1}{8}\log|9-4x^2| + c$

6, $\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\,dx = \frac{1}{2}\int \frac{2e^{2x} - 2e^{-2x}}{e^{2x} + e^{-2x}}\,dx$

$= \frac{1}{2}\log|e^{2x} + e^{-2x}| + c$

7, $\int \frac{e^{2x}-1}{e^{2x}+1}\,dx = \int \frac{e^{x}[e^{x}-e^{-x}]}{e^{x}[e^{x}+e^{-x}]}\,dx$

$= \int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\,dx$

$= \log|e^{x}+e^{-x}|$

8, $\int \frac{1}{x+x\log x}\,dx = \int \frac{1}{x(1+\log x)}\,dx$

$= \int \frac{1/x}{1+\log x}\,dx$

$= \log|1+\log x| + c$

9, $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$= -\int \frac{-\sin x}{\cos x}\,dx$

$= -\log|\cos x| + c$

$= \log|\sec x| + c$

10, $\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$

$= \log|\sin x| + c$

11, $\int \sec x \, dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} dx$

$$= \int \frac{\sec^2 x + \sec x \cdot \tan x}{\sec x + \tan x} dx$$

$$= \log|\sec x + \tan x| + c$$

12, $\int \operatorname{cosec} x \, dx = \int \frac{\operatorname{cosec} x(\operatorname{cosec} x - \cot x)}{\operatorname{cosec} x - \cot x} dx$

$$= \int \frac{\operatorname{cosec}^2 x - \operatorname{cosec} x \cdot \cot x}{\operatorname{cosec} x - \cot x} dx$$

$$= \log|\operatorname{cosec} x - \cot x| + c$$