

Integrals class-9

Type-7

$$\int \frac{dx}{ax^2+bx+c} \quad \text{or} \quad \int \frac{dx}{\sqrt{ax^2+bx+c}} \quad \text{or} \quad \int \sqrt{ax^2+bx+c} dx$$

1, Find $\int \frac{1}{x^2-6x+13} dx$

$$\int \frac{1}{x^2-6x+13} dx = \int \frac{1}{x^2-6x+9-9+13} dx$$

$$= \int \frac{1}{(x-3)^2+4} dx$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{x-3}{2} \right) + c$$

2, Find $\int \frac{1}{\sqrt{x^2-6x+13}} dx$

$$\int \frac{1}{\sqrt{x^2-6x+13}} dx = \int \frac{1}{\sqrt{x^2-6x+9-9+13}} dx$$

$$= \int \frac{1}{\sqrt{(x-3)^2+4}} dx$$

$$= \log \left| x-3 + \sqrt{(x-3)^2+4} \right| + c$$

3, Find $\int \sqrt{x^2+4x-5} dx$

$$\int \sqrt{x^2+4x-5} dx = \int \sqrt{x^2+4x+4-4-5} dx$$

$$= \int \sqrt{(x+2)^2-9} dx$$

$$= \frac{x+2}{2} \sqrt{(x+2)^2-9} - \frac{9}{2} \log |x+2+\sqrt{(x+2)^2-9}| + C$$

4, $\int \frac{dx}{9x^2+6x+5} = \int \frac{dx}{9(x^2+\frac{2}{3}x+\frac{5}{9})}$

$$= \frac{1}{9} \int \frac{dx}{x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}+\frac{5}{9}}$$

$$= \frac{1}{9} \int \frac{dx}{(x+\frac{1}{3})^2+(\frac{2}{3})^2}$$

$$= \frac{1}{9} \cdot \frac{1}{2/3} \tan^{-1} \left(\frac{x+\frac{1}{3}}{2/3} \right) + C$$

$$= \frac{1}{6} \tan^{-1} \left(\frac{3x+1}{2} \right) + C$$

5, $\int \sqrt{3-2x-x^2} dx = \int \sqrt{-(x^2+2x-3)} dx$

$$= \int \sqrt{-(x^2+2x+1-1-3)} dx$$

$$= \int \sqrt{-[(x+1)^2 - 4]} dx$$

$$= \int \sqrt{(2)^2 - (x+1)^2} dx$$

$$= \frac{x+1}{2} \sqrt{4 - (x+1)^2} + \frac{4}{2} \sin^{-1}\left(\frac{x+1}{2}\right) + C$$

6, Find $\int \frac{dx}{\sqrt{7-6x-x^2}}$

$$\int \frac{dx}{\sqrt{7-6x-x^2}} = \int \frac{dx}{\sqrt{-[x^2+6x-7]}}$$

$$= \int \frac{dx}{\sqrt{-[x^2+6x+9-9-7]}}$$

$$= \int \frac{dx}{\sqrt{-[(x+3)^2 - 16]}}$$

$$= \int \frac{dx}{\sqrt{4^2 - (x+3)^2}}$$

$$= \sin^{-1}\left(\frac{x+3}{4}\right) + C$$