Integrals class-10

Type-8

$$\int \frac{px+q}{ax^2+bx+c}dx \text{ or } \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx \text{ or } \int (px+q)\sqrt{ax^2+bx+c}\,dx$$

Put $px+q = A\frac{d}{dx}(ax^2+bx+c)+B$

(1) Find $\int \frac{5x+3}{x^2+4x+10}dx$

Let $5x+3 = A(2x+4)+B$

Eq: coeff: of x	Eq: constants
$5 = 2A$	$3 = 4A+B$
$5/2 = A$	$3 = 4\times\frac{5}{2}+B$
	$B = -7$

$$\therefore \int \frac{5x+3}{x^2+4x+10}dx = \int \frac{5/2(2x+4)+-7}{x^2+4x+10}dx$$

$$= 5/2\int \frac{2x+4}{x^2+4x+10}dx - 7\int \frac{1}{x^2+4x+10}dx$$

$$= \frac{5}{2}\log|x^2+4x+10| - 7\int \frac{1}{(x+2)^2+(\sqrt{6})^2}dx$$

$$= \frac{5}{2}\log|x^2+4x+10| - 7\times\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{x+2}{\sqrt{6}}\right)+c$$

(2) Find $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$

$$\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$$

$5x+3 = A(2x+4)+B$

Eq: coeff: of x

$5 = 2A$

$\frac{5}{2} = A$

Eq: constants

$3 = 4A+B$

$3 = 4\times\frac{5}{2}+B$

$B = -7$

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx = \int \frac{\frac{5}{2}(2x+4)+(-7)}{\sqrt{x^2+4x+10}}\,dx$$

$$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}\,dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}\,dx$$

$$= \frac{5}{2}+2\sqrt{x^2+4x+10} - 7\int \frac{1}{\sqrt{(x+2)^2+(\sqrt{6})^2}}\,dx$$

$$= 5\sqrt{x^2+4x+10} - 7\cdot \log\left|x+2+\sqrt{(x+2)^2+6}\right| + c$$

(3) $\int \frac{4x+1}{\sqrt{2x^2+x-3}}\,dx$

$$\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)}$$

$$= 2\sqrt{2x^2+x-3} + c$$

(4) Find $\int \frac{x+2}{\sqrt{x^2-1}}\,dx$

$$\int \frac{x+2}{\sqrt{x^2-1}}\,dx = \int \frac{x}{\sqrt{x^2-1}}\,dx + \int \frac{2}{\sqrt{x^2-1}}\,dx$$

$$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}\,dx + 2\int \frac{1}{\sqrt{x^2-1}}\,dx$$

$$= \frac{1}{2}\times 2\sqrt{x^2-1} + 2\log\left|x+\sqrt{x^2-1}\right| + c$$

(5) Find $\int x\sqrt{x^2+x}\,dx$

Let $x = A(2x+1)+B$

Eq: coeff: of x

$1 = 2A$

$\frac{1}{2} = A$

Eq: constants

$0 = A+B$

$B = -\frac{1}{2}$

$$\int x\sqrt{x^2+x}\,dx = \int \left[\frac{1}{2}(2x+1) - \frac{1}{2}\right]\sqrt{x^2+x}\,dx$$

$$= \frac{1}{2}\int (2x+1)\sqrt{x^2+x}\,dx - \frac{1}{2}\int \sqrt{x^2+x}\,dx$$

$$= \frac{1}{2}\cdot \frac{(x^2+x)^{3/2}}{3/2} - \frac{1}{2}\int \sqrt{x^2+x+\frac{1}{4}-\frac{1}{4}}\,dx$$

$$= \frac{1}{3}(x^2+x)^{3/2} - \frac{1}{2}\int \sqrt{(x+\frac{1}{2})^2-(\frac{1}{2})^2}\,dx$$

$$= \frac{1}{3}(x^2+x)^{3/2} - \frac{1}{2}\left[\frac{x+\frac{1}{2}}{2}\sqrt{(x+1/2)^2-\frac{1}{4}}\right.$$

$$\left. - \frac{1/4}{2}\log\left|x+\frac{1}{2}+\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}\right|+c\right.$$

$$= \frac{1}{3}(x^2+x)^{3/2} - \frac{(2x+1)}{8}\sqrt{(x+1/2)^2-\frac{1}{4}}$$

$$+\frac{1}{16}\log\left|x+\frac{1}{2}+\sqrt{(x+1/2)^2-\frac{1}{4}}\right|+c$$