Integrals Class-16

Evaluation of Definite integral by Substitution.

1, Find $\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx$

$$\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx = \int_0^{\pi/4} u \cdot du$$

$\tan^{-1}x = u$

$\frac{1}{1+x^2}dx = du$

$x=0, u=0$

$x=1, u=\pi/4$

$$= \left[\frac{u^2}{2}\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[\frac{\pi^2}{16}\right] = \frac{\pi^2}{32}$$

2, Find $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$

$$\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$$

$\cos x = u$

$-\sin x\,dx = du$

$x=0, u=1$

$x=\pi/2; u=0$

$$= \int_1^0 \frac{-du}{1+u^2}$$

$$= -\left[\tan^{-1}u\right]_1^0 = -\left[0-\tan^{-1}(1)\right]$$

$$= \pi/4$$

3, Find $\int_0^{\pi/4} \sin^3 2x \cdot \cos 2x \, dx$

$$\int_0^{\pi/4} \sin^3 2x \cdot \cos 2x \, dx$$

$\sin 2x = u$

$2\cos 2x \, dx = du$

$x = 0, u = 0$

$x = \pi/4, u = 1$

$$= \int_0^1 u^3 \cdot \frac{du}{2}$$

$$= \frac{1}{2}\left[\frac{u^4}{4}\right]_0^1 = \frac{1}{8}[1-0] = \frac{1}{8}$$

4, Find $\int_0^1 x \cdot e^{x^2} dx$

$$\int_0^1 x e^{x^2} dx$$

$x^2 = u$

$2x\,dx = du$

$x\,dx = du/2$

$x = 0, u = 0$

$x = 1, u = 1$

$$= \int_0^1 e^u \cdot \frac{du}{2}$$

$$= \frac{1}{2}[e^u] = \frac{1}{2}[e-1]$$

5, Find $\int_{-1}^1 5x^4 \sqrt{x^5+1} \, dx$

$x^5 + 1 = u$

$5x^4 dx = du$

$x = -1, u = 0$

$x = 1, u = 2$

$$\int_{-1}^{1} 5x^4\sqrt{x^5+1}\,dx = \int_{0}^{2}\sqrt{u}\,du$$

$$= \left[\frac{u^{3/2}}{3/2}\right]_0^2 = \frac{2}{3}\left[2^{3/2}\right]$$

$$= \frac{2}{3}\left[2\sqrt{2}\right] = \underline{\underline{\frac{4\sqrt{2}}{3}}}$$

6/ Find $\displaystyle\int_0^{\pi/2} \sqrt{\sin x}\;\underline{\cdot\cos^5 x}\,dx$

$$\int_0^{\pi/2}\sqrt{\sin x}\,\cos^5 x\,dx = \int_0^{\pi/2}\sqrt{\sin x}\cdot(\cos^2 x)^2\cos x\,dx$$

$$= \int_0^{\pi/2}\sqrt{\sin x}\,(1-\sin^2 x)^2\cdot\cos x\,dx$$

$\sin x = u$

$\cos x\,dx = du$

$x = 0,\ u = 0$

$x = \pi/2,\ u = 1$

$$= \int_0^1 \sqrt{u}\,(1-u^2)^2\,du$$

$$= \int_0^1 \sqrt{u}\left[1-2u^2+u^4\right]du$$

$$= \int_0^1\left(\sqrt{u} - 2u^{5/2} + u^{9/2}\right)du$$

$$= \left[\frac{2}{3}u^{3/2} - 4\frac{u^{7/2}}{7} + 2\frac{u^{11/2}}{11}\right]_0^1 = \left[\frac{2}{3} - \frac{4}{7} + \frac{2}{11}\right]$$

$$= \frac{64}{231}\ //$$