More Questions and Solutions

3 mark question

14. Find the domain and range of the function

$$f(x) = \frac{1}{\sqrt{x-7}}.$$

Soln: $f(x) = \frac{1}{\sqrt{x-7}}$

$f(x)$ is defined if $x - 7 > 0 \Rightarrow x > 7$

$\therefore$ Domain $= (7, \infty)$

Let $f(x) = y$

then $y = \frac{1}{\sqrt{x-7}}$ $\quad \therefore \sqrt{x-7} = \frac{1}{y}$

$x - 7 = \frac{1}{y^2}$, $x = \frac{1}{y^2} + 7$

$x \in (7, \infty) \Rightarrow y \in R^+$

Hence range $= R^+$

4 mark question

15. Consider the functions $f(x) = \sqrt{x-2}$,

$$g(x) = \frac{x+1}{x^2-2x+1}.$$

(i) Find the domain of f (ii) Find the domain of g

(iii) $(f+g)$ x (iv) $(fg)(x)$

Soln: (i) $f(x) = \sqrt{x-2}$ and $g(x) = \frac{x+1}{x^2-2x+1}$

Domain of f is $x - 2 \geq 0$. ie $x \geq 2$.

(ii) $x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1$

$\therefore$ Domain of g is $R - \{1\}$

(iii) $(f+g)(x) = f(x) + g(x) = \sqrt{x-2} + \frac{x+1}{x^2-2x+1}$

(iv) $(fg)(x) = f(x) . g(x)$

$= \sqrt{x-2} . \frac{x+1}{x^2-2x+1} = \frac{(x+1)\sqrt{x-2}}{x^2-2x+1}$

6 mark question

16. (i) If $f(x) = ax + b$ where a and b are integers $f(-1) = -5$ and $f(3) = 3$ then find a and b.

(ii) If $(2a + b, a - b) = (8, 3)$ then find a and b.

Soln: (i) $f(x) = ax + b$

$f(-1) = -5 \Rightarrow a(-1) + b = -5$

$\Rightarrow -a + b = -5$ -------- (1)

$f(3) = 3 \quad \Rightarrow a(3) + b = 3$

$\Rightarrow 3a + b = 3$ ----- (2)

$(2) - (1) \rightarrow 4a = 8$

$a = 2$, $\therefore b = -5 + a = -5 + 2 = -3$

$\therefore a = 2, b = -3$

(ii) $(2a + b, a - b) = (8, 3)$

$2a + b = 8$, $a - b = 3$

Adding, $3a = 11$

$\therefore a = \frac{11}{3}$

$\therefore b = a - 3 = \frac{11}{3} - 3 = \frac{2}{3}$

$\therefore a = \frac{11}{3}, b = \frac{2}{3}$

1. The relation *f* is defined by $f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$

The relation *g* is defined by

$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

Show that f is a function and g is not a function.

Soln: Given $f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$

For the values $0 \leq x < 3$ and $3 < x \leq 10$, $f(x)$ is uniquely defined.

When $x = 3$, $x^2 = 3^2 = 9$ and $3x = 3 \times 3 = 9$.

$\therefore$ f(3) is also uniquely defined.

$\Rightarrow$ f(x) is uniquely defined for all values of x such that $0 \le x \le 10$. $\therefore$ f is a function.

Also given $g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$

For the values of x, such that $0 \le x < 2$ and $2 < x \le 10$, g(x) is uniquely defined.

When $x = 2$, $x^2 = 4$ and $3x = 3 \times 2 = 6$.

ie, 2 has two images in g.

$\therefore$ g(2) is not uniquely defined. $\therefore$ g is not a function.

2. If $f(x) = x^2$, find $\dfrac{f(1.1)-f(1)}{(1.1-1)}$.

Soln: Given $f(x) = x^2$

$$\therefore \frac{f(1.1)-f(1)}{(1.1-1)} = \frac{(1.1)^2-1^2}{(1.1-1)} = \frac{1.21-1}{0.1} = \frac{0.21}{0.1}$$

$$= \frac{21}{10} = 2.1$$

3. Find the domain of the function

$$f(x) = \frac{x^2+2x+1}{x^2-8x+12}$$

Soln: Given $f(x) = \dfrac{x^2+2x+1}{x^2-8x+12}$.

f(x) is not defined for the values of x for which $x^2 - 8x + 12 = 0$.

$x^2 - 8x + 12 = 0 \Rightarrow (x-2)(x-6) = 0$

$\Rightarrow x = 2$ or $x = 6$.

$\therefore$ f(x) is defined for all real values of x other than $x = 2$ and $x = 6$.

$\therefore$ Domain of $f = R - \{2, 6\}$.

4. Find the domain and the range of the real function f defined by $f(x) = \sqrt{x-1}$.

Soln: Given $f(x) = \sqrt{x-1}$.

f(x) is defined for all values of x such that

$x - 1 \ge 0 \Rightarrow x \ge 1$.

$\therefore$ Domain of f = set of all real values $\ge 1 = [1, \infty)$

Here $y = f(x) = \sqrt{x-1}$

When $x = 1$, $y = 0$.

Also for all $x > 1$, $y > 0$

$\therefore$ Range of $f = [0, \infty)$ = set of non-negative reals.

5. Find the domain and the range of the real function f defined by $f(x) = |x - 1|$.

Soln: Given $f(x) = |x - 1|$.

Since the modulus of a real number is uniquely defined for all real values of x, domain of f = set of real numbers = R.

Since modulus of a real number is a non-negative real number, range of f = set of non-negative real numbers $= [0, \infty)$

6. Let $f = \left\{\left(x, \dfrac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R into R. Determine the range of f.

Soln: Given $f(x) = \dfrac{x^2}{1+x^2}$.

Clearly, f(x) is defined for all real values of x.

$\therefore$ Domain of f = R

Here $y = \dfrac{x^2}{1+x^2} \Rightarrow (1 + x^2)\, y = x^2 \Rightarrow y + x^2y = x^2$

$\Rightarrow x^2(1-y) = y \Rightarrow x^2 = \dfrac{y}{1-y}$. $\therefore \dfrac{y}{1-y} \ge 0$

When $y = 0$, $\dfrac{y}{1-y} = 0$

Also when $y = 1$, $\dfrac{y}{1-y}$ is not defined.

$\therefore$ We cannot take the value $y = 1$.

Now for all other values of $y > 1$, $\dfrac{y}{1-y}$ is negative.

$\therefore 0 \le y < 1$

$\therefore$ Range of $y = [0, 1)$.

7. Let f, g : R $\to$ R be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\dfrac{f}{g}$.

Soln: Given f, g : R $\to$ R is defined by $f(x) = x + 1$ and $g(x) = 2x - 3$.

$\therefore f + g$, $f - g$ and $\dfrac{f}{g}$ are functions from R $\to$ R.

We have $(f + g)(x) = f(x) + g(x)$

$= x + 1 + 2x - 3 = 3x - 2$, for all $x \in R$.

$(f - g)(x) = f(x) - g(x)$

$= x + 1 - (2x - 3) = 4 - x$, for all $x \in R$.

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x+1}{2x-3}$, for all $x \in R - \{x : 2x - 3 = 0\}$

ie, for all $x \in R - \left\{\dfrac{3}{2}\right\}$.