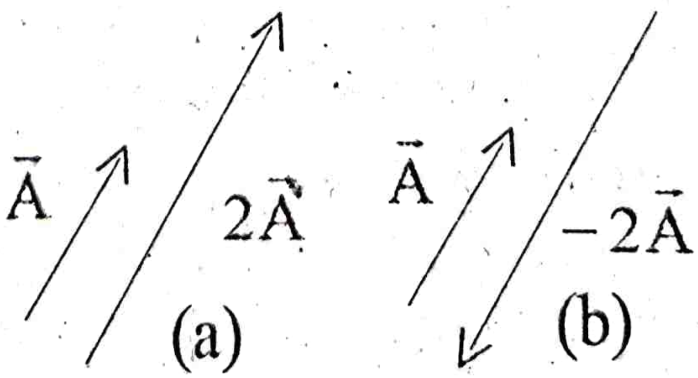
4.3. Multiplication of vectors by real numbers

The multiplication of a vector $\vec{A}$ by a real number n is another vector $n\vec{A}$ of magnitude n times and direction same or opposite as that of $\vec{A}$, depending upon whether n is a positive or negative real number.

$\left|n\vec{A}\right| = n\left|\vec{A}\right|$

In fig (a) n = 2 and in (b) n = –2



- **Vector algebra**

Consider some displacement vectors.

1. $\vec{A} = 4m$ East, $\vec{B} = 3m$ East

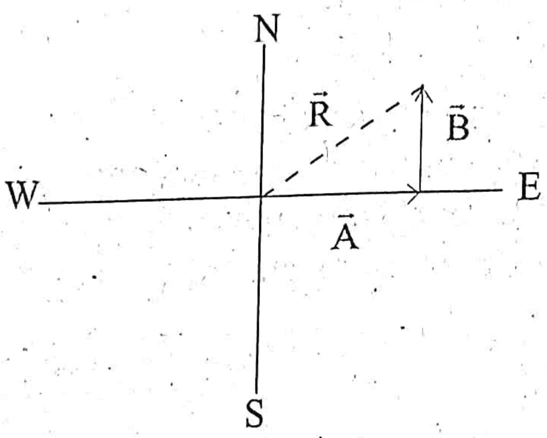
   Then $\vec{R} = \vec{A} + \vec{B} = 4m$ East + 3m East = 7m East

2. $\vec{A} = 4m$ East, $\vec{B} = 3m$ West

Then $\vec{R} = \vec{A} + \vec{B} = 1m$ East

3. $\vec{A} = 4m$ East, $\vec{B} = 3m$ North

Then $\vec{R} = \vec{A} + \vec{B} = 5m$ NE



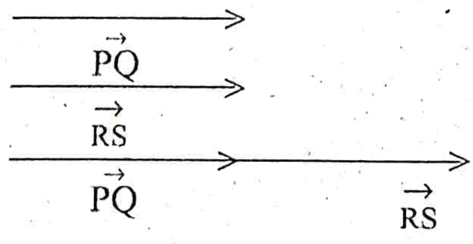
$\vec{A}$ and $\vec{B}$ are identical in the above three cases, but the magnitude of resultants are different. This is because of the direction of the physical quantity. Since the vectors possess directions in addition to their magnitudes, they cannot be added by simple laws of algebra applicable to scalars. Therefore we use vector algebra for addition, subtraction and multiplication of vectors.

[**Note:** One vector cannot be divided by another vector. That is vector division is not defined.]

4.4. Addition and subtraction of vectors- Graphical method

(a) When two vectors are in same direction

If two vectors are in same direction, their resultant is equal to the sum of their magnitudes. The direction of resultant being the same as that of original vectors. (രണ്ട് വെക്ടറുകൾ ഒരേ ദിശയിലാണെങ്കിൽ അവയുടെ പരിണതഫലം രണ്ടിന്റെയും ആകെ തുകയായിരിക്കും. ദിശ മുൻപുള്ള വെക്ടറുകളുടേതുതന്നെ.)

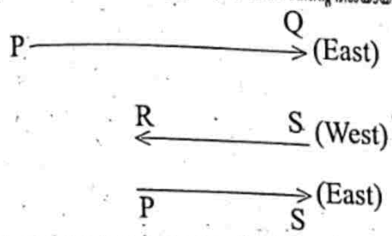


$$\overrightarrow{PQ} + \overrightarrow{RS} = \overrightarrow{PS}$$

(b) When two vectors are acting in opposite direction.

If two vectors are in opposite direction, their resultant is equal to the difference of their magnitudes; the direction of resultant being the same as that of the larger vector. (രണ്ടു വെക്ടറുകൾ വിപരീത ദിശയിലാണെങ്കിൽ അവ തമ്മിലുള്ള

വ്യത്യാസമാണ് പരിണതഫലം. മൂല്യം കൂടിയ വെക്ടറിന്റെ ദിശയായിരിക്കും അതിനുള്ളത്.)



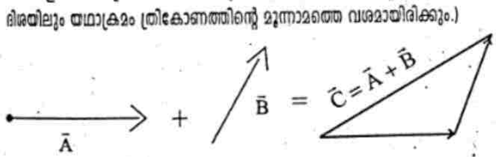
(c) **When two vectors are inclined to each other.**

If two vectors are inclined to each other then their resultant can be found by triangle, parallelogram and polygon law of vector addition. (രണ്ട് വെക്ടറുകൾ പരസ്പരം ചെരിഞ്ഞതാണെങ്കിൽ ട്രയാംഗിൾ, പാരലലൊഗ്രം, പോളിഗൺ എന്നീ വെക്ടർ സങ്കലന നിയമം ഉപയോഗിച്ച് പരിണതഫലം കണ്ടുപിടിക്കാം.)

The resultant of two (or more) vectors is a single vector, which produces the same effect as the individual vectors produce together.

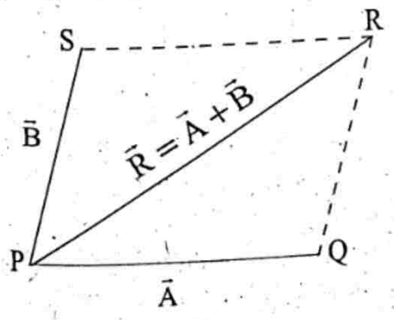
Triangle law of vector addition

It states that if two vectors can be represented in magnitude and direction by the two sides of a triangle taken in the same order then the resultant is represented completely (both in magnitude and direction) by the third side of the triangle taken in the order. (രണ്ടു വെക്ടറുകളെ അതിന്റെ മൂല്യത്തിലും ദിശയിലും ഒരു ത്രികോണത്തിന്റെ രണ്ടു വശങ്ങളായി യഥാക്രമം കണക്കാക്കുകയാണെങ്കിൽ, അതിന്റെ പരിണതഫലം മൂല്യത്തിലും ദിശയിലും യഥാക്രമം ത്രികോണത്തിന്റെ മൂന്നാമത്തെ വശമായിരിക്കും.)



Parallelogram law of vector addition

It states that if two vectors acting simultaneously at a point can be represented both in magnitude and direction by the two adjacent sides of a parallelogram, the resultant is represented completely (both in magnitude and direction) by the diagonal of the parallelogram passing through that point. (ഒരു ബിന്ദുവിലെ രണ്ട് വെക്ടറുകളെ ഒരേ സമയം മൂല്യത്തിലും ദിശയിലും ഒരു പാരലലോഗ്രാമിന്റെ അടുത്ത രണ്ടുവശങ്ങളായി യഥാക്രമം കണക്കാക്കുകയാണെങ്കിൽ അതിന്റെ പരിണിതഫലം മൂല്യത്തിലും ദിശയിലും പ്രസ്തുത ബിന്ദുവിൽകൂടി പോകുന്ന കർണ്ണത്തിനു തുല്യമായിരിക്കും.)



Note: Since in a parallelogram, the opposite sides are equal and parallel, they must represent equal vectors.

Therefore $\vec{SR} = \vec{PQ} = \vec{A}$ and $\vec{QR} = \vec{PS} = \vec{B}$

In triangle PQR, sides $\vec{PQ}$ and $\vec{QR}$ represent the vectors $\vec{A}$ and $\vec{B}$ taken in order. Therefore according to triangle law of vector addition,

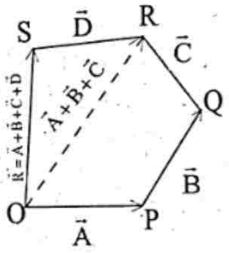
$\vec{PQ} + \vec{QR} = \vec{PR}$

ie, $\vec{A} + \vec{B} = \vec{R}$

That is the parallelogram law of vector addition follows the triangle law of vector addition.

Polygon law of vector addition

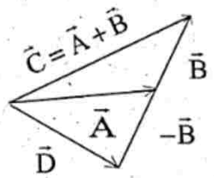
It states that if a number of vectors are represented in magnitude and direction by the sides of a polygon taken in the same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in the opposite order.



This law is also just the extension of the triangle law of addition of vectors.

Subtraction of vectors

Vector subtraction is nothing but the addition of a reverse vector. To subtract one vector from another, the direction of the vector to be subtracted is reversed and then the non- reversed vector and the 'reversed vector' are added by the usual vector addition method.



$\vec{D} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$

$\vec{C} = \vec{A} + \vec{B}$

Zero vectors

A vector whose magnitude is zero is known as zero (or null) vector. The direction of zero vector is not defined. The zero vector is denoted by $\vec{0}$. The following two operations lead to zero vector.

1. When a vector is multiplied by zero, the resultant is zero vector. $0 \times |\vec{A}| = \vec{0}$
2. When the negative of a vector is added to the vector, the resultant is zero vector. Thus

   $\vec{A} + (-\vec{A}) = \vec{0}$
3. The result of adding a zero vector to any vector is the vector itself $\vec{A} + \vec{0} = \vec{A}$