2.5 Introduction to Boolean algebra.

The name Boolean Algebra is given to honour the British mathematician George Boole.

Boolean algebra deals with two states true or false otherwise Yes or No and numerically either 0 or 1.

2.5.1 Binary valued quantities

A logical decision which gives YES or No values is a binary decision, A statement which gives YES or NO values(TRUE or FALSE) is a logical statement or truth function. A variable which can assign TRUE or FALSE (1 or 0) values is a logical variable.

2.5.2 Boolean operators and logic gates

Logical Operators are AND, OR and NOT.

A logical gate is a physical device (electronic circuit)that can perform logical operations on one or more logical inputs and produce a single logical output.

A table represents the set f all possible values and the corresponding results in a statement is called truth table.

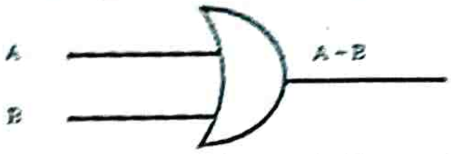
a. The OR operator and OR gate

The OR operator gives a 1 either one of the operands is 1. If both operands are 0, it produces 0.

The truth table of X OR Y is

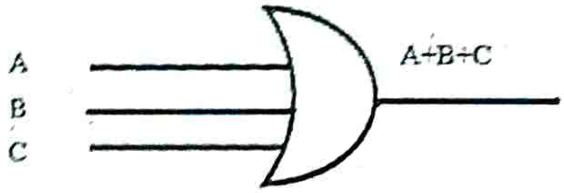
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

The logical OR gate is given below.



The truth table and the gate for the Boolean expression Y=A+B+C

A	B	C	A+B+C
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



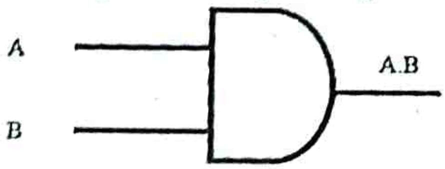
b. The AND operator and AND gate

The AND operator gives a 1 if and only if both operands are 1. If either one of the operands is 0, it produces 0

The truth table of X AND Y is

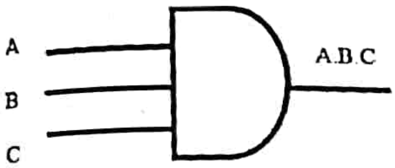
A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

The logical AND gate is given below.



The truth table and the gate for the Boolean expression Y=A.B.C

A	B	C	A.B.C
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



C. The NOT operator and NOT gate
It produces the vice versa. NOT gate is also called inverter. It is a unary operator that means it has only one input and one output.
The truth table of NOT X is

A	NOT A
0	1
1	0

The logical NOT gate is given below.

2.6 Basic postulates of Boolean algebra.

Boolean algebra consists of some fundamental laws. These laws are called postulates.
Postulate 1 : Principles of 0 and 1
If A ≠ 0 , then A=1 and A 1, then A=0
Postulate 2: OR Operation (Logical Addition)
0+0=0 0+1=1 1+0=1 1+1=1
Postulate 3: AND Operation (Logical Multiplication)
0.0=0 0.1=0 1.0=0 1.1=1
Postulate 4: NOT Operation(Logical Negation or Compliment Rule)
$\overline{0}=1$ $\overline{1}=0$

Principle of Duality
When changing the OR(+) to AND(.), AND (.) to OR(+), 0 to 1 and 1 to 0 in a Boolean expression we will get another Boolean relation which is the dual of the first, this is the principle of duality.

2.7 Basic theorems of Boolean algebra

There are some standard and accepted rules in every theory, these rules are known as axioms of the theory.

2.7.1 Identity law

If X is a Boolean variable, the law states that
(i) 0 + X = X
(ii) 1+X =1 (these are additive identity law)
(iii) 0.X=0
(iv) 1.X=X (these are multiplicative identity law)
Following are the truth tables

0	X	0+X
0	0	0
0	1	1

1	X	1+X
1	0	1
1	1	1

0	X	0.X
0	0	0
0	1	0

1	X	1.X
1	0	0
1	1	1

2.7.2 Idempotent law

This law states that
I) X + X = X ii) X . X = X

X	X	X+X
0	0	0
1	1	1

X	X	X.X
0	0	0
1	1	1

2.7.3 Involution law

This law states that

$$\overline{\overline{X}} = X$$

The compliment of compliment of a number is the number itself.

X	$\overline{X}$	$\overline{\overline{X}}$
0	1	0
1	0	1

2.7.4 Complimentary law

This law states that

(i) $X+\overline{X}=1$

(ii) $X.\overline{X}=0$

The truth table is given below

X	$\overline{X}$	$X+\overline{X}$
0	1	1
1	0	1

X	$\overline{X}$	$X.\overline{X}$
0	1	0
1	0	0

2.7.5 Commutative law

This law allows to change the position of variable in OR and AND
i) X + Y = Y + X
ii) X . Y = Y . X
The truth table is given below

X	Y	X+Y	Y+X
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

X	Y	X.Y	Y.X
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

2.7.6 Associative law

It allows grouping of variables differently
i) X+(Y+Z) = (X+Y) + Z
ii) X.(Y.Z) = (X.Y).Z
The truth table is given below

X	Y	Z	X+Y	Y+X	X+(Y+Z)	(X+Y)+Z
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

X	Y	Z	X.Y	Y.X	X.(Y.Z)	(X.Y).Z
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

2.7.7 Distributive law

This law allows expansion of multiplication over addition and also allows addition operation over multiplication.

i) X.(Y+Z)=X.Y+X.Z

ii) X+Y.Z=(X+Y).(X+Z)

The truth table is given below

X	Y	Z	Y+Z	X.(Y+Z)	X.Y	X.Z	X.Y+X.Z
0	0		0	0	0	0	0
0	0		1	0	0	0	0
0	1		0	0	0	0	0
0	1		1	0	0	0	0
1	0		0	0	0	0	0
1	0		1	1	0	1	1
1	1		0	1	1	0	1
1	1		1	1	1	1	1

X	Y	Z	Y.Z	X+Y.Z	X+Y	X+Z	(X+Y).(X+Z)
0	0		0	0	0	0	0
0	0		1	0	0	0	0
0	1		0	0	0	1	0
0	1		1	1	1	1	1
1	0		0	0	1	1	1
1	0		1	1	1	1	1
1	1		0	1	1	1	1
1	1		1	1	1	1	1

2.7.8 Absorption law

It is a kind of distributive law in which two variables are used and result will be one of them

i) X+(X.Y) = X ii) X.(X+Y)=X

The truth table is given below

X	Y	X.Y	X+(X.Y)
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

X	Y	X.+Y	X.(X+Y)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

2.8 De Morgan's theorem

Demorgan's first theorem states that

$$\overline{X+Y}=\overline{X}\,.\,\overline{Y}$$

ie. the compliment of sum of two variables equals product of their compliments.

The second theorem states that

$$\overline{X\,.\,Y}=\overline{X}+\overline{Y}$$

ie. The compliment of the product of two variables equals the sum of the compliment of that variables.