

## TEXTBOOK EXERCISES 3.2

1.  $\cos x = \frac{-1}{2}$

We have,  $\sin^2 x = 1 - \cos^2 x = 1 - \frac{1}{4} = \frac{3}{4}$

$$\Rightarrow \sin x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

Since  $x$  is in IIIrd quadrant,  $\sin x$  is negative.

$$\therefore \sin x = \frac{-\sqrt{3}}{2} \quad \therefore \tan x = \frac{\sin x}{\cos x} = \frac{-\sqrt{3}}{\frac{-1}{2}} = \sqrt{3}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{-2}{\sqrt{3}}$$

$$\sec x = \frac{1}{\cos x} = -2, \quad \cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

2.  $\sin x = \frac{3}{5}$ ,  $x$  is in IInd quadrant.

$$\therefore \cos^2 x = 1 - \sin^2 x = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm \frac{4}{5}$$

Since  $x$  is in IInd quadrant,  $\cos x$  is negative.

$$\therefore \cos x = \frac{-4}{5}$$

$$\tan x = \frac{-3}{4}, \quad \operatorname{cosec} x = \frac{5}{3}, \quad \sec x = \frac{-5}{4}, \quad \cot x = \frac{-4}{3}$$

3.  $\cot x = \frac{3}{4}$ ,  $x$  lies in third quadrant.

We have,  $\operatorname{cosec}^2 x - 1 = \cot^2 x$

$$\therefore \operatorname{cosec}^2 x = 1 + \frac{9}{16} = \frac{25}{16} \Rightarrow \operatorname{cosec} x = \pm \frac{5}{4}$$

$$\operatorname{cosec} x = \frac{-5}{4}; \text{ since } x \text{ is in third quadrant.}$$

$$\therefore \sin x = \frac{-4}{5}$$

$$\cos x = \cot x \cdot \sin x = \frac{3}{4} \cdot \frac{-4}{5} = \frac{-3}{5}$$

$$\sec x = \frac{-5}{3}, \quad \tan x = \frac{4}{3}$$

4.  $\sec x = \frac{13}{5}$ ,  $x$  is in IVth quadrant.

$$\therefore 1 + \tan^2 x = \sec^2 x \Rightarrow \tan^2 x = \sec^2 x - 1$$

$$\Rightarrow \tan^2 x = \frac{169}{25} - 1 = \frac{144}{25}$$

$$\therefore \tan x = \pm \frac{12}{5}$$

But  $\tan x = \frac{-12}{5}$ , since  $x$  is in the IVth quadrant.

$$\sec x = \frac{13}{5} \Rightarrow \cos x = \frac{5}{13}$$

$$\sin x = \tan x \cdot \cos x = \frac{-12}{5} \cdot \frac{5}{13} = \frac{-12}{13}$$

$$\operatorname{cosec} x = \frac{-13}{12}, \quad \cot x = \frac{-5}{12}$$

5.  $\tan x = \frac{-5}{12}$ ,  $x$  is in IInd quadrant.

$$\therefore \sec^2 x = 1 + \tan^2 x = 1 + \frac{25}{144} = \frac{169}{144}$$

$$\Rightarrow \sec x = \pm \frac{13}{12}$$

But  $\sec x = \frac{-13}{12}$ , since  $x$  is in II<sup>nd</sup> quadrant.

$$\therefore \cos x = \frac{-12}{13}$$

$$\sin x = \tan x \cos x = \frac{-5}{12} \times \frac{-12}{13} = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{13}{5}, \quad \cot x = \frac{-12}{5}$$

6.  $\sin 765^\circ = \sin(2 \times 360^\circ + 45^\circ) = \sin(4\pi + 45^\circ)$

$$= \sin 45^\circ = \frac{1}{\sqrt{2}}$$

7.  $\operatorname{cosec}(-1410^\circ) = -\operatorname{cosec}(1410^\circ)$

$$(\because \operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta)$$

$$\begin{aligned}
&= -\operatorname{cosec}(3 \times 360^\circ + 330^\circ) = -\operatorname{cosec}(6\pi + 330^\circ) \\
&= -\operatorname{cosec}330^\circ = -\operatorname{cosec}(360^\circ - 30^\circ) \\
&= -(-\operatorname{cosec}30^\circ) = \operatorname{cosec}30^\circ = 2.
\end{aligned}$$

$$8. \tan\left(\frac{19\pi}{3}\right) = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}.$$

$$\begin{aligned}
9. \sin\left(\frac{-11\pi}{3}\right) &= -\sin\left(\frac{11\pi}{3}\right) = -\sin\left(2\pi + \frac{5\pi}{3}\right) \\
&= -\sin\left(\frac{5\pi}{3}\right) = -\sin\left(2\pi - \frac{\pi}{3}\right)
\end{aligned}$$

$$= -\left(-\sin\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

$$10. \cot\left(\frac{-15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right) = -\cot\left(2\pi + \frac{7\pi}{4}\right)$$

$$= -\cot\left(\frac{7\pi}{4}\right) = -\cot\left(\pi + \frac{3\pi}{4}\right)$$

$$= -\cot\frac{3\pi}{4} = -\cot\left(\pi - \frac{\pi}{4}\right) = -\left(-\cot\frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1$$

### Formula for $(\frac{\pi}{2} - x)$

1.  $\sin(\frac{\pi}{2} - x) = \cos x$
2.  $\cos(\frac{\pi}{2} - x) = \sin x$
3.  $\tan(\frac{\pi}{2} - x) = \cot x$
4.  $\operatorname{cosec}(\frac{\pi}{2} - x) = \sec x$
5.  $\sec(\frac{\pi}{2} - x) = \operatorname{cosec} x$
6.  $\cot(\frac{\pi}{2} - x) = \tan x$

### Formula for $(\frac{\pi}{2} + x)$

1.  $\sin(\frac{\pi}{2} + x) = \cos x$
2.  $\cos(\frac{\pi}{2} + x) = -\sin x$
3.  $\tan(\frac{\pi}{2} + x) = -\cot x$
4.  $\operatorname{cosec}(\frac{\pi}{2} + x) = \sec x$
5.  $\sec(\frac{\pi}{2} + x) = -\operatorname{cosec} x$
6.  $\cot(\frac{\pi}{2} + x) = -\tan x$

### Formula for $(\pi - x)$

1.  $\sin(\pi - x) = \sin x$
2.  $\cos(\pi - x) = -\cos x$
3.  $\tan(\pi - x) = -\tan x$
4.  $\operatorname{cosec}(\pi - x) = \operatorname{cosec} x$
5.  $\sec(\pi - x) = -\sec x$
6.  $\cot(\pi - x) = -\cot x$

### Formula for $(\pi + x)$

1.  $\sin(\pi + x) = -\sin x$
2.  $\cos(\pi + x) = -\cos x$
3.  $\tan(\pi + x) = \tan x$
4.  $\operatorname{cosec}(\pi + x) = -\operatorname{cosec} x$
5.  $\sec(\pi + x) = -\sec x$
6.  $\cot(\pi + x) = \cot x$

### Formula for $(\frac{3\pi}{2} - x)$

1.  $\sin(\frac{3\pi}{2} - x) = -\cos x$
2.  $\cos(\frac{3\pi}{2} - x) = -\sin x$
3.  $\tan(\frac{3\pi}{2} - x) = \cot x$
4.  $\operatorname{cosec}(\frac{3\pi}{2} - x) = -\sec x$
5.  $\sec(\frac{3\pi}{2} - x) = -\operatorname{cosec} x$
6.  $\cot(\frac{3\pi}{2} - x) = \tan x$

### Formula for $(\frac{3\pi}{2} + x)$

1.  $\sin(\frac{3\pi}{2} + x) = -\cos x$
2.  $\cos(\frac{3\pi}{2} + x) = \sin x$
3.  $\tan(\frac{3\pi}{2} + x) = -\cot x$
4.  $\operatorname{cosec}(\frac{3\pi}{2} + x) = -\sec x$
5.  $\sec(\frac{3\pi}{2} + x) = \operatorname{cosec} x$
6.  $\cot(\frac{3\pi}{2} + x) = -\cot x$

### Formula for $(2\pi - x)$

1.  $\sin(2\pi - x) = -\sin x$
2.  $\cos(2\pi - x) = \cos x$
3.  $\tan(2\pi - x) = -\tan x$
4.  $\operatorname{cosec}(2\pi - x) = -\operatorname{cosec} x$
5.  $\sec(2\pi - x) = \sec x$
6.  $\cot(2\pi - x) = -\cot x$

### Formula for $(2\pi + x)$

1.  $\sin(2\pi + x) = \sin x$
2.  $\cos(2\pi + x) = \cos x$
3.  $\tan(2\pi + x) = \tan x$
4.  $\operatorname{cosec}(2\pi + x) = \operatorname{cosec} x$
5.  $\sec(2\pi + x) = \sec x$
6.  $\cot(2\pi + x) = \cot x$

## Sign of Trigonometric functions

$$\sin -\theta = -\sin \theta$$

$$\cos -\theta = \cos \theta$$

$$\tan -\theta = -\tan \theta$$

