

 **Points to Remember**

Statement	Algebra
◆ Two more than a number	$x + 2$
◆ Two less than a number	$x - 2$
◆ Two times of a number	$2x$
◆ Half of a number	$\frac{x}{2}$ or $\frac{1}{2}x$
◆ Two consecutive natural numbers	$x, x + 1$
◆ Two consecutive even numbers	$x, x + 2$
◆ Two consecutive odd numbers	$x, x + 2$
◆ A number and its reciprocal	$x, \frac{1}{x}$
◆ Two consecutive term of an arithmetic sequence with common difference 'd'	$x, x + d$
◆ Square of a number	$x^2$
◆ Two numbers with sum 15	$x, 15 - x$
◆ Two numbers with difference 15	$x, x + 15$ or $x, x - 15$
◆ Two numbers with product 15	$x, \frac{15}{x}$
◆ $(x + a)^2 = x^2 + 2ax + a^2$	
◆ $(x - a)^2 = x^2 - 2ax + a^2$	
◆ Method of solving second degree equations of the form	
◆ $(x + a)^2 = b^2$ OR $(x - a)^2 = b^2$	
◆ $x + a = \pm b$	$(x - a)^2 = b^2$
◆ $x + a \pm b$	$x - a = \pm b$
◆ $x + a = b$ Or $x + a = -b$	$x - a = b$ Or $x - a = -b$
◆ $x = b - a$ Or $x = -b - a$	$x = b + a$ Or $x = -b + a$

## MATHEMATICS

- ◆ To convert  $x^2 + 2ax$  into a perfect square, we have add the square of half of the coefficient of  $x$

Example :  $x^2 + 2x = 24$

Here the coefficient of  $x$  is 2

Square of half of it is  $1^2$

$$x^2 + 2x + 1^2 = 24 + 1^2$$

$$(x + 1)^2 = 25$$

$$(x + 1)^2 = 5^2$$

$$x + 1 = \pm 5$$

$$x + 1 = 5 \quad \text{Or} \quad x + 1 = -5$$

$$x = 5 - 1 \quad \quad \quad x = -5 - 1$$

$$= 4 \quad \quad \quad = -6$$

- ◆ The general form of a second degree equation is  $ax^2 + bx + c = 0$ ,  $a \neq 0$

Two solutions are  $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$  and  $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

### Worksheet 1

Complete the following table

Statement	Equation	Solution
a) 12 added to a number gives 40. find the number.	$x + 12 = 40$	$x = 40 - 12 = 28$
b) 12 subtracted from a number gives to	$x - \dots = \dots$	
c)	$x + 3 = 14$	
d) 7 subtracted from, 3 times of a number is 20. Find the number ?		
e) 5 added to one fourth of a number is 10. Find the number?		
f) Square of a number is 100. Find the number?		
g)	$x^2 = 36$	
h)	$x^2 = 10$	
i) Square of $x + 3$ is 36. Find $x$ ?	$(x + 3)^2 = 36$	$x + 3 = \pm 6$ $x + 3 = 6$ Or $x + 3 = -6$ $x = 6 - 3$ Or $x = -6 - 3$ $x = 3$ $x = -9$

j) Square of $x + 8$ is 81. Find $x$ ?		
k) The value of $x^2 + 4x + 4$ is 25. Find $x$	$x^2 + 4x + 4 = 25$ $(x + 2)^2 = 25$	
l) The value of $x^2 - 14x + 49$ is 64. Find $x$ ?	$(x - 7)^2 = 64$	
m) The value of $x^2 - 6x + 9$ is 100. Find $x$ ?		

From the above table, write down the first degree equations

Write down the second degree equations.

**Worksheet 2**

Each side of a square is increased by 4 cm, its area becomes 625 cm<sup>2</sup>.  
What is the side of original square?

One side of original square =  $x$

One side of new square =  $x + \square$

Area of new square =  $(x + \square)^2$

$$(x + \square)^2 = 625$$

$$x + \square = \sqrt{625}$$

$$x + \square = 25$$

$$x = \square - \square$$

$$= \square$$

∴ One side of the original square =  $\square$  cm.

**Worksheet 3**

Length of a rectangle is 10 cm more than its breadth and its area is 200 cm<sup>2</sup>. Find its length and breadth?

Breadth =  $x$

Length =  $x + \square$

Area =  $x(x + \square)$

$$x(x + \square) = 200$$

$$x^2 + \square = 200$$

$$x^2 + 10x + \square = 200 + \square$$

$$(x + \square)^2 = \square$$

$$x + \square = \square$$

$$x = \square - \square$$

$$= \square$$

∴ Breadth of the rectangle =  $\square$  cm

Length of the rectangle =  $\square + 10$

$$= \square \text{ cm}$$

**Worksheet 4**

In the semicircle ACB, PC is  $\perp$  to AB. Length of AP is 12 cm more than PB and PC = 8 cm. Find the diameter of the circle.

$$PB = x$$

$$PA = x + \square$$

$$PA \times PB = PC^2$$

$$(x + \square) x = 8^2$$

$$x^2 + 12x + \square = 8^2 + \square$$

$$(x + \square)^2 = \square$$

$$x + \square = \square$$

$$x = \square - \square$$

$$= \square$$

$$\therefore PB = \square \text{ cm}$$

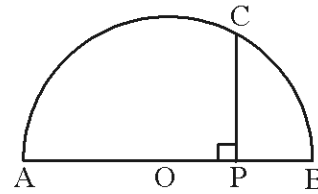
$$PA = \square + 12$$

$$= \square \text{ cm}$$

$$\text{Diameter} = PA + PB$$

$$= \square + \square$$

$$= \square \text{ cm}$$



**Worksheet 5**

Hypotenuse of a right triangle is 2 less than 2 times its base and the third side is 2 more than its base. Find the length of the sides?

$$\text{Base} = x$$

$$\text{Hypotenuse} = 2x - \square$$

$$\text{Third side} = x + \square$$

$$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$$

$$x^2 + (x + \square)^2 = (2x - \square)^2$$

$$x^2 + x^2 + \square x + \square = \square x^2 - \square x - \square = 0$$

$$2x^2 + \square x + \square - \square x^2 + \square x - \square = 0$$

$$-2x^2 + \square x = 0$$

$$2x^2 - \square x = 0$$

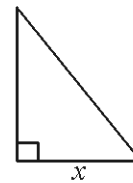
$$x^2 - \square x = 0$$

$$x^2 - \square x + \square = 0 + \square$$

$$(x - \square)^2 = \square$$

$$x - \square = \square$$

$$x = \square + \square$$



$$= \square$$

$$\text{Base} = \square \text{ cm}$$

$$\text{Hypotenuse} = 2 \square - 2$$

$$= 2 \square - 2$$

$$= \square \text{ cm}$$

$$\text{Third side} = \square + 2$$

$$= \square \text{ cm}$$

**Worksheet 6**

Complete the following table.

Equations	Completing the square	Solutions
$x^2 + 10x = 24$	$x^2 + 10x + 5^2 = 24 + 5^2$ $x^2 + 10x + 25 = 49$ $(x + 5)^2 = 49$	$x + 5 = \pm 7$ $x = \pm 7 - 5$ $x = 2, x = -12$
$x^2 + 8x = 20$		
$x^2 - 12x = 28$		
$x^2 - 6x - 16 = 0$		
$x^2 + 3x = 40$		
$x^2 - 2x = 5$		

**Worksheet 7**

How many terms of the arithmetic sequence 7, 9, 11, ..... must be added to get 40?

7, 9, 11, .....

first term, f =  $\square$

common difference, d =  $\square$

sum = 40

$$\frac{d}{2}n^2 + \left[ f - \frac{d}{2} \right]n = 40$$

$$\frac{\square}{2}n^2 + \left[ \square - \frac{\square}{2} \right]n = 40$$

$$\square n^2 + \square n = 40$$

$$\square n^2 + \square n - 40 = 0$$

$$a = \square, b = \square, c = \square$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned}
 x &= \frac{-\square \pm \sqrt{\square^2 - 4 \times \square \times \square}}{2 \times \square} \\
 &= \frac{-\square \pm \sqrt{\square - \square}}{2 \times \square} \\
 &= \frac{-\square \pm \sqrt{\square}}{\square} \\
 &= \frac{-\square + \square}{\square}, \frac{-\square - \square}{\square} \\
 x &= \square, \square
 \end{aligned}$$

ie, sum of first  $\square$  terms of the sequence 7, 9, 11, ..... is 40.

**Worksheet 8**

The perimeter of a rectangle is 82 cm and its area is 400 square cm. Find it's sides.

Breadth of the rectangle be  $x$ .

$$\text{Perimeter} = 82 \text{ cm.}$$

$$2 (\text{length} + \text{breadth}) = 82$$

$$2 (\text{length} + x) = 82$$

$$\text{length} + x = \frac{82}{2} = 41$$

$$\text{length} = 41 - x$$

Area of the rectangle = 400 sq. cm

$$\text{length} \times \text{breadth} = 400$$

$$(41 - \square) \times x = 400$$

$$41 \square - \square^2 = 400$$

$$-\square^2 + 41\square = 400$$

$$-\square^2 + 41\square - 400 = 0$$

$$\square^2 + 41\square + 400 = 0$$

$$a = 1, b = -41, c = 400$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \times \square \times \square}}{2 \times \square}$$

$$= \frac{\square \pm \sqrt{\square^2 - \square}}{2 \times \square}$$

$$= \frac{\square \pm \sqrt{\square}}{\square}$$

$$= \frac{\square + \square}{\square}, \frac{\square + \square}{\square}$$

$$= \frac{\square}{\square}, \frac{\square}{\square}$$

$$= \square, \square$$

Breadth =  $\square$  cm, Length =  $\square$  cm

## ANSWERS

### Worksheet 1

- a.  $x + 12 = 40, x = 40 - 12 = 28$
- b.  $x - 12 = 26; x = 26 + 12 = 38$
- c. 3 added to a number gets 16. What is the number?  
 $x = 14 - 3 = 11$
- d.  $3x - 7 = 20; \quad 3x = 20 + 7$   
 $3x = 27, x = \frac{27}{3} = 9$
- e.  $\frac{x}{4} + 5 = 10; \quad \frac{x}{4} = 10 - 5$   
 $\frac{x}{4} = 5, \quad x = 5 \times 4 = 20$
- f.  $x^2 = 100; \quad x = 10, -10$
- g. The square of a number is 36. What is the number?  
 $x = 6, -6.$
- h. The square of a number is 10. What is the number?  
 $x = \sqrt{10}, -\sqrt{10}$
- j.  $(x + 8)^2 = 81 \quad \quad \quad x + 8 = 9 \quad \quad \quad x + 8 = -9$   
 $\quad \quad \quad \quad \quad \quad \quad \quad x = 9 - 8 \quad \quad \quad x = -9 - 8$   
 $\quad \quad \quad \quad \quad \quad \quad \quad x = 1 \quad \quad \quad x = -17$
- k.  $x + 2 = 5 \quad \quad \quad \text{or } x + 2 = -5$   
 $x = 5 - 2 = 3 \quad \quad \quad \text{or } x = -5 - 2 = -7$
- l.  $x - 7 = 8 \quad \quad \quad \text{or } x - 7 = -8$   
 $x = 8 + 7 \quad \quad \quad x = -8 + 7$   
 $x = 15 \quad \quad \quad x = -1$

## MATHEMATICS

m.  $(x - 3)^2 = 100$  ;

$$x - 3 = \pm 10$$

$$x - 3 = -10$$

$$x - 3 = 10$$

$$x = -10 + 3$$

$$x = 10 + 3$$

$$x = -7$$

$$x = 13$$

n.  $x + 12 = 40$

$$x - 12 = 26$$

$$x + 3 = 14$$

$$3x - 7 = 20$$

$$\frac{x}{4} + 5 = 10$$

o.  $x^2 = 100$

$$x^2 = 36$$

$$x^2 = 10$$

$$(x + 3)^2 = 36$$

$$(x + 8)^2 = 81$$

$$x^2 + 4x + 4 = 25$$

$$(x - 7)^2 = 64$$

$$(x - 3)^2 = 100$$

### Worksheet 2

One side of the original square =  $x$

One side of the new square =  $x + 4$

Area of the new square =  $(x + 4)^2$

$$(x + 4)^2 = 625$$

$$x + 4 = \sqrt{625}$$

$$x + 4 = 25$$

$$x = 25 - 4 = 21$$

One side of the original square = 21 cm

### Worksheet 3

Breadth =  $x$

Length =  $x + 10$

Area =  $(x + 10) \times x$

$$x(x + 10) = 200$$

$$x^2 + 10x + 25 = 200 + 25$$



$$(x + 5)^2 = 225$$

$$x + 5 = 15$$

$$x = 15 - 5 = 10$$

$$\text{Breadth} = 10 \text{ cm}$$

$$\text{Length} = 10 + 10 = 20 \text{ cm}$$

#### Worksheet 4

$$PB = x \text{ cm}$$

$$PA = x + 12$$

$$PA \times PB = PC^2$$

$$(x + 12) \times x = 8^2$$

$$x(x + 12) = 64$$

$$x^2 + 12x = 64$$

$$x^2 + 12x + 36 = 64 + 36$$

$$(x + 6)^2 = 100$$

$$(x + 6) = \sqrt{100}$$

$$x + 6 = 10$$

$$x = 10 - 6 = 4$$

$$PB = 4 \text{ cm}$$

$$PA = 4 + 12 = 16 \text{ cm}$$

$$\text{Diameter} = 4 + 16 = 20 \text{ cm}$$

#### Worksheet 5

$$\text{Base} = x$$

$$\text{Hypotenuse} = 2x - 2$$

$$\text{Altitude} = x + 2$$

$$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$$

$$x^2 + (x + 2)^2 = (2x - 2)^2$$

$$x^2 + x^2 + 4x + 4 = 4x^2 - 8x + 4$$

$$2x^2 + 4x + 4 - 4x^2 + 8x - 4 = 0$$

$$-2x^2 + 12x = 0$$

$$2x^2 - 12x = 0$$

$$x^2 - 6x = 0$$

$$x^2 - 6x + 9 = 9$$

$$(x - 3)^2 = 3^2$$

$$(x - 3) = 3$$

$$x = 3 + 3 = 6$$

$$\text{Base} = 6 \text{ cm}$$

$$\text{Hypotenuse} = 2 \times 3 - 2$$

$$= 6 - 2 = 4 \text{ cm}$$

$$\text{Altitude} = 3 + 2 = 5 \text{ cm}$$

**Worksheet 6**

b.  $x^2 + 8x + 4^2 = 20 + 4^2$

$$x^2 + 8x + 16 = 20 + 16$$

$$(x + 4)^2 = 36$$

$$(x + 4) = \pm 6$$

$$x + 4 = \pm 6$$

$$x = \pm 6 - 4$$

$$x = 2, -10$$

c.  $x^2 - 12x + 6^2 = 28 + 6^2$

$$(x - 6)^2 = 64$$

$$x - 6 = \pm \sqrt{64}$$

$$x - 6 = \pm 8$$

$$x = \pm 8 + 6$$

$$x = 14, -2$$

d.  $x^2 - 6x - 16 = 0$

$$x^2 - 6x + 9 = 16 + 9$$

$$(x - 3)^2 = 25$$

$$x - 3 = \pm 5$$

$$x = \pm 5 + 3$$

$$x = 8, -2$$

e.  $x^2 + 3x = 40$

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = 40 + \left(\frac{3}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 = 40 + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{169}{4}$$

$$x + \frac{3}{2} = \pm \frac{13}{2}$$

$$x = \frac{\pm 13}{2} - \frac{3}{2}$$

$$x = 5, -8$$

f.  $x^2 - 2x = 5$

$$x^2 - 2x + 1 = 5 + 1$$

$$(x - 1)^2 = 6$$

$$x - 1 = \pm \sqrt{6}$$

$$x = \pm \sqrt{6} + 1$$

$$x = \sqrt{6} + 1, -\sqrt{6} + 1$$

**Worksheet 7**

7, 9, 11, .....

First term,  $f = 7$

common difference,  $d = 2$

sum = 40

$$\frac{d}{2}n^2 + \left[f - \frac{d}{2}\right]n = 40$$

$$\frac{2}{2}n^2 + \left[7 - \frac{2}{2}\right]n = 40$$

$$1n^2 + (7 - 1)n = 40$$

$$n^2 + 6n = 40$$

$$n^2 + 6n - 40 = 0$$

$$a = 1, b = 6, c = -40$$

$$\begin{aligned} n &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times -40}}{2 \times 1} \\ &= \frac{-6 \pm \sqrt{36 + 160}}{2} \\ &= \frac{-6 \pm \sqrt{196}}{2} \\ &= \frac{-6 \pm 14}{2}, \quad \frac{-6 - 14}{2} \\ &= \frac{8}{2}, \quad \frac{-20}{2} \\ &= 4, -10 \end{aligned}$$

The sum of first 4 terms of the arithmetic sequence is 40.

**Worksheet 8**

$$\text{Breadth} = x \text{ cm}$$

$$\text{Perimeter} = 82 \text{ cm}$$

$$2 (\text{length} + \text{breadth}) = 82$$

$$2 (\text{length} + x) = 82$$

$$\text{length} + x = \frac{82}{2} = 41$$

$$\text{length} = 41 - x$$

$$\text{Area} = 400 \text{ sq. cm}$$

$$\text{length} \times \text{breadth} = 400$$

$$(41 - x) \times x = 400$$

$$41x - x^2 = 400$$

$$-x^2 + 41x - 400 = 0$$

$$x^2 - 41x + 400 = 0$$

$$a = 1, b = -41, c = 400$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned}
 x &= \frac{-(-41) \pm \sqrt{(-41)^2 - 4 \times 1 \times 400}}{2 \times 1} \\
 &= \frac{41 \pm \sqrt{1681 - 1600}}{2} \\
 &= \frac{41 \pm \sqrt{81}}{2} \\
 &= \frac{41 \pm 9}{2}, \quad \frac{41 - 9}{2} \\
 &= \frac{50}{2}, \quad \frac{32}{2} \\
 &= 25, 16
 \end{aligned}$$

Breadth = 16 cm, Length = 25 cm

